One Dimensional Waves in a Cube

1 Introduction

The main objectives of this Demo Model are to

- Demonstrate the ability of Coustyx to model a Cube with a vibrating face using Indirect model and solve for the acoustic field point pressure distribution inside the Cube.
- Derive analytical solutions for the interior (bounded) problem of the cube based on one dimensional wave equation.
- Validate Coustyx software by comparing the results from Coustyx to the analytical solutions.

2 Model description

We model a cube of size $1\,\text{m} \times 1\,\text{m} \times 1\,\text{m}$. The fluid medium in and around the cube is air with sound speed $c = 343\,\text{m/s}$ and mean density $\rho_o = 1.21\,\text{kg/m}^3$. The characteristic impedance of air $Z_o = \rho_o c = 415.03\,Rayl$. The wavenumber at a frequency ω is given as $k = \omega/c$. The BE mesh of the cube is shown in Figure 1.

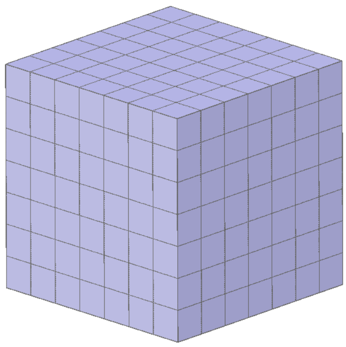

Figure 1: Boundary element mesh for a cube.

The cube face at $y = 0$ is vibrating with unit velocity in $+y$ direction, that is $\tilde{v}_y = 1\,\text{m/s}$. Since the mesh normal is in $-y$ direction, the boundary condition applied on the face $y = 0$ is $v_n = -\tilde{v}_y = -1\,\text{m/s}$. In the Coustyx model, this boundary condition is applied as an "Uniform Normal Velocity of Continuous type", that is, $v_n^+ = v_n^- = v_n$, where $+$ and $-$ correspond to the sides of the mesh in the same and opposite directions of the mesh normal. All the other faces of the cube are assumed to be rigid, that is v_n=0.

The BE mesh has quadratic coordinate connectivity as well as quadratic variable node connectivity. Coustyx indirect BE method is used to solve the exterior and interior problems simultaneously. Since we have analytical solutions only for the cube interior problem, we compare field point pressures inside the cube from both Coustyx and analytical solutions.

3 Analytical solution

The cube interior problem is analytically solved using 1-D plane wave assumptions.

The solution to a general 1-D wave equation is of the form

$$\tilde{p}(y) = Ae^{iky} + Be^{-iky} \tag{1}$$

Assume the following general boundary conditions (with the rest of the faces of the cube assumed to be rigid)

$$\begin{aligned} \tilde{v}_y &= v_{yo}, y = 0 \\ \tilde{v}_y &= v_{yl}, y = l \end{aligned} \tag{2}$$

The pressure at any point in the plane at y is given by

$$\tilde{p}(y) = \frac{i\rho c(v_{yo} \cos k(l-y) - v_{yl} \cos ky)}{\sin kl} \tag{3}$$

The velocity in $+y$ direction is

$$\tilde{v}_y = \frac{v_{yo} \sin k(l-y) + v_{yl} \sin ky}{\sin kl} \tag{4}$$

4 Results and validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx Indirect model. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. In the demo model, the analysis is performed at a frequency $f = 54.59Hz$ using the Fast Multipole Method (FMM) by running "Run Validation - FMM". Coustyx analysis results, along with the analytical solutions, are written to the output file "validation_results_fmm.txt". The results can be plotted using the matlab file "PlotResults.m".

For the demo model, the boundary conditions applied are, $v_{yo} = 1$ and $v_{yl} = 0$. The indirect BE model solves for the surface potentials μ and σ. These are, in turn, used to compute field pressures at the interior points in the cube. Figure 2 shows the comparison of field point pressures at points (0.5,y,0.5) obtained from Coustyx and analytical solutions. The comparisons show very good agreement between these two solutions.

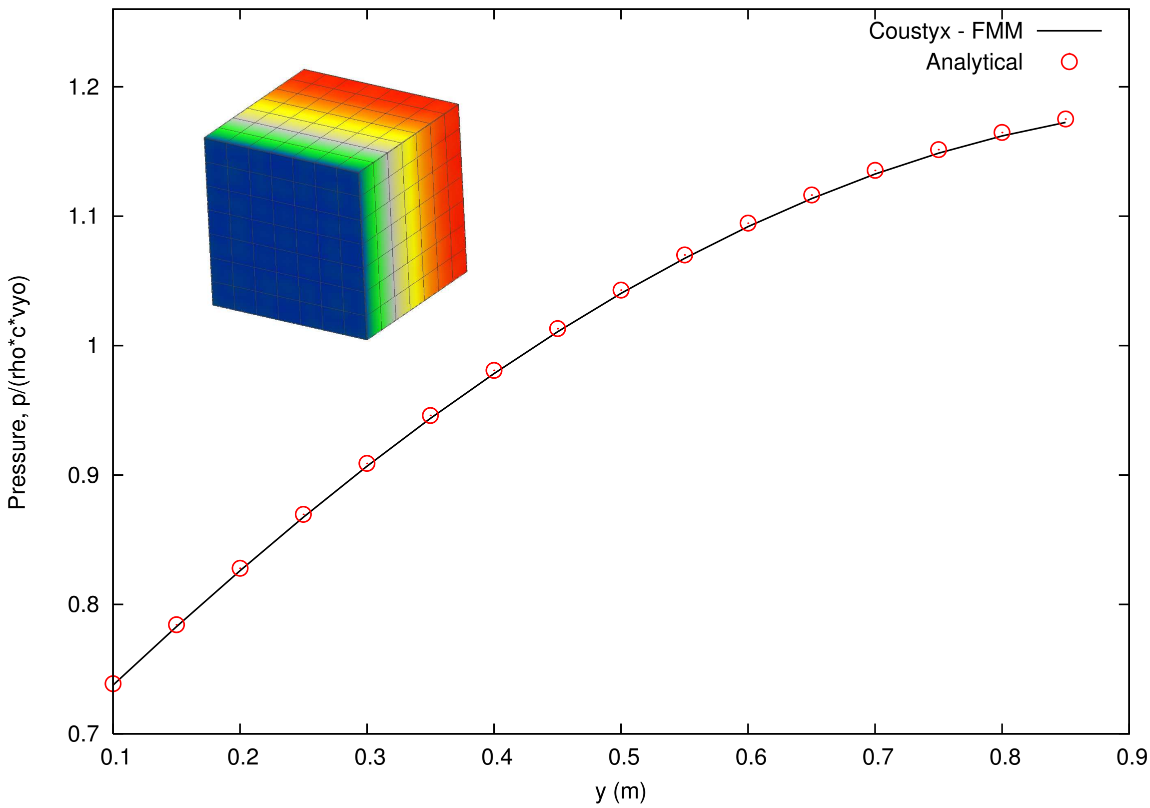

Figure 2: Field pressure comparisons at points (0.5,y,0.5) from Coustyx and analytical methods.