Acoustic Radiation from a Transversely Oscillating Circular Disk

1 Introduction

The main objectives of this Demo Model are

- Demonstrate the ability of Coustyx to model a transversely oscillating disk using Indirect model and solve for its radiated acoustic field.
- Validate Coustyx software by comparing the results from Coustyx to the published numerical solution computed from variational method by Wu et al. [1].

2 Model description

We model a circular disk of radius $a = 1\,\mathrm{m}$. The thickness of the disk is assumed to be negligible. The fluid medium surrounding the disk is air with sound speed $c = 343\,\mathrm{m/s}$ and mean density $\rho_o = 1.21\,\mathrm{kg/m^3}$. The characteristic impedance of air $Z_o = \rho_o c = 415.03\,Rayl$. The wavenumber at a frequency ω is given as $k = \omega/c$. The disk is oscillating with a unit transverse velocity, perpendicular to the plane of the disk, in the $+z$ direction; that is $v_z = 1\,\mathrm{m/s}$. The BE mesh of the sphere is shown in Figure 1.

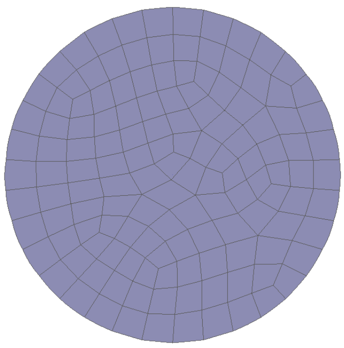

Figure 1: Boundary element mesh for a disk of unit radius.

The BE mesh has quadratic coordinate connectivity as well as quadratic variable node connectivity. Coustyx indirect BE method is used to solve this problem.

3 Boundary Conditions

The boundary condition applied on the disk is an "Uniform Normal Velocity of Continuous type", that is, $v_n^+ = v_n^- = v_n$, where $+$ and $-$ correspond to the sides of the mesh in the same and opposite directions of the mesh normal. Since the mesh normal is in the $-z$ direction, the boundary condition applied in the Coustyx model is $v_n = -v_z = -1\,\mathrm{m/s}$.

4 Numerical method - Published results

In this method [1], the variational formulation derived for acoustic radiation and diffraction problems is solved using Rayleigh-Ritz method. The basis functions are selected such that they satisfy certain characteristics of the exact solution for a disk. For example, the tangential derivative of the surface pressure is infinite at the edge of the disk and this knowledge is used in selecting appropriate basis functions. Refer to Wu et al. [1] for details.

5 Results and Validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx Indirect model. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. In the demo model, the analysis is performed for a frequency range of $f = 54.59Hz - 5 * 54.59Hz$ in steps of $\Delta f = 54.59Hz$, that is $ka = 1, 2, 3, 4, 5$, using the Fast Multipole Method (FMM) by running "Run Validation - FMM". Coustyx analysis results, along with the analytical solutions, are written to the output file "validation_results_fmm.txt". The results can be plotted using the matlab file "PlotResults.m".
The indirect BE model solves for the surface potentials μ and σ. These are, in turn, used to compute surface pressures and field pressures. For this example problem, the surface pressures can be directly computed from μs as follows:
From the definition of double-layer potential we have

$$\tilde{p}(p^+) - \tilde{p}(p^-) = \mu(p) \tag{1}$$

and,

$$\frac{\tilde{p}(p^+) + \tilde{p}(p^-)}{2} = \int_S \mu(q) \frac{\partial G_H(q,p)}{\partial n_q} ds(q) = 0 \quad \forall p \in S \tag{2}$$

where $G_H(q, p)$ is the free-space Green's function, $+$ and $-$ correspond to the sides of the mesh in the same and opposite directions of the mesh normal.
The rhs of the equation 2 above reduces to the zero as $\frac{\partial G_H(q,p)}{\partial n_q}$=0 for the points p and q lying on the disk plane. From above equations, the pressures on both sides of the surface can be evaluated to $\tilde{p}(p^+) = \frac{\mu}{2}$ and $\tilde{p}(p^-) = -\frac{\mu}{2}$.
Figure 2 shows comparisons of the surface pressures from Coustyx analysis and from the results extracted from the numerical solutions published by Wu et al. [1]. The dimensionless surface pressures $\frac{p}{\rho_o c v_z}$ are plotted against dimensionless r/a for various $ka = 1, ..5$, where r is the distance from the center of the disk. At the free edge, the pressures reduce to zero and the slopes of the curves tend to infinity. Coustyx solutions capture these characteristics well, and show good comparisons with the published numerical results.

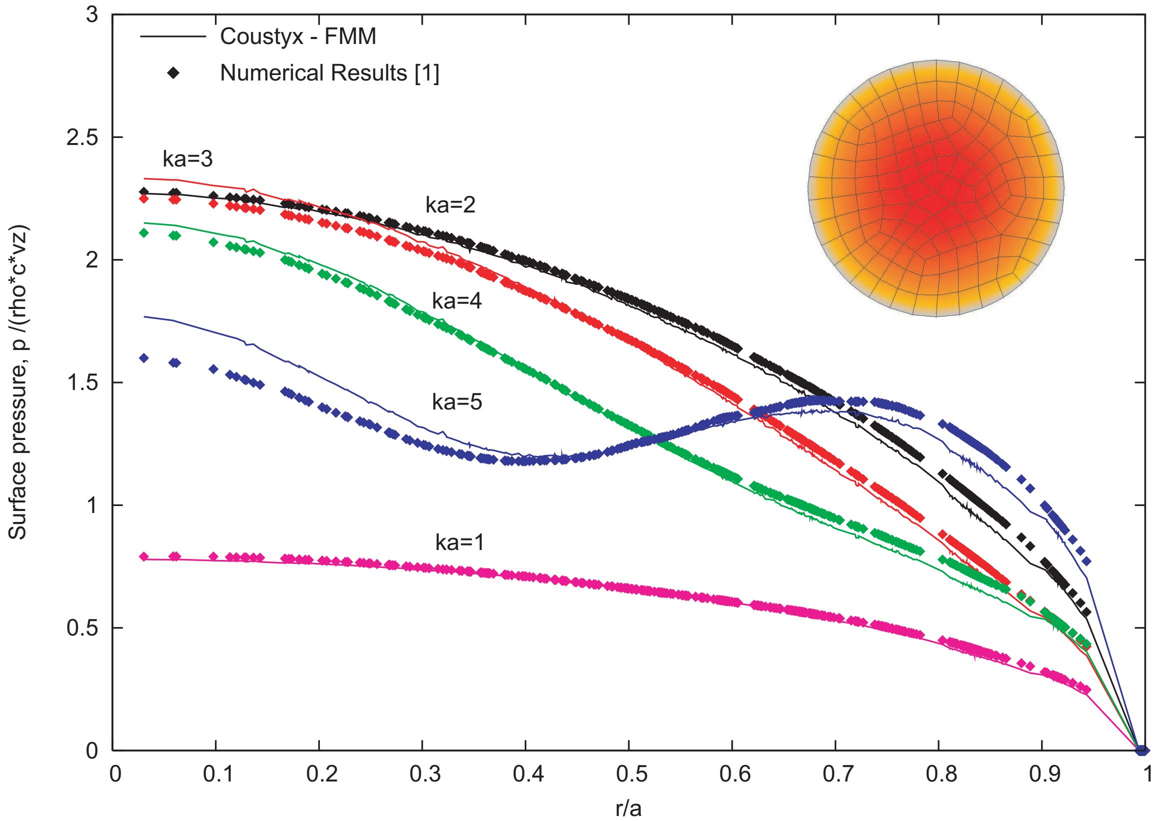

Figure 2: Comparisons of pressures from Coustyx and published numerical results for various values of ka. Solid lines indicate Coustyx results; dots indicate published results.

References

[1] X. F. Wu, A. D. Pierce, and J. H. Ginsberg. Variational method for computing surface acoustic pressure on vibrating bodies, applied to transversely oscillating disks. *IEEE Journal of Oceanic Engineering*, 12:412–418, 1987.