# Muffler Transmission Loss – Simple Expansion Chamber

## 1 Introduction

The main objectives of this Demo Model are

- Demonstrate the ability of Coustyx to model a muffler using Indirect model and solve the acoustics problem to compute the transmission loss for the muffler.
- Derive four-pole parameters from Coustyx analysis and use them to compute the transmission loss.
- Validate Coustyx software by comparing the transmission loss computed from Coustyx to the analytical solution and published experimental measurements by Tao and Seybert [1].

## 2 Model description

In this example we model a simple expansion chamber and compute the transmission loss. The BEA solutions are compared with the experiment results extracted from the publication by Tao and Seybert [1].

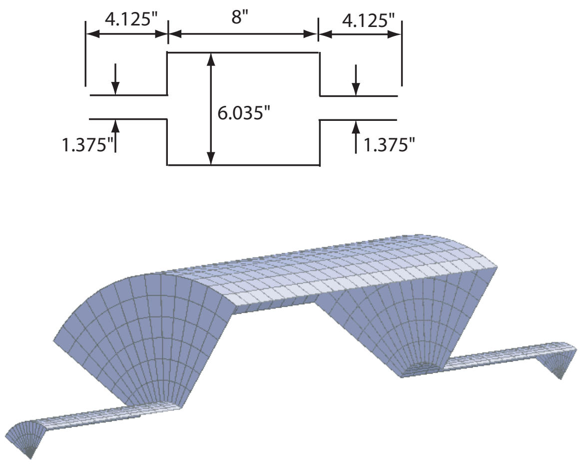


Figure 1: Boundary element mesh for the simple expansion chamber.

Only a quarter of the muffler is modeled to take advantage of the symmetry of the system and reduce the size of the problem. Figure 1 shows BEA mesh for the quarter model of the muffler. The dimensions of the muffler are given in inches. The fluid medium inside and around the muffler is air with sound speed $c = 13503.937\,\text{inch/s}(343\,\text{m/s})$ and mean density $\rho_o = 4.3714e{-}5\,\text{lb/inch}^3(1.21\,\text{kg/m}^3)$. Note that the units for the speed of sound and mean density are chosen to be consistent with the units for the length dimensions of the mesh. The characteristic impedance of air is $Z_o = \rho_o c$. The wavenumber at a frequency $\omega$ is $k = \omega/c$.

The BE mesh has linear coordinate connectivity as well as linear variable node connectivity. Coustyx indirect BE method is used to solve this problem.

## 3 Boundary Conditions

Transmission loss is computed from the four-pole parameters derived running Coustyx. The four-pole parameters are part of the transfer matrix connecting inlet and outlet pressures and velocities (more details explained in the next section). The following two cases with different boundary conditions are run to compute the four-pole parameters:

**Configuration *a* or Case 1** The boundary conditions are,

1. inlet: Uniform Normal Velocity of Continuous type, $v_n = -1$.
2. outlet: Uniform Normal Velocity of Continuous type, $v_n = 0$.
3. The rest of the muffler is assumed to be rigid.

**Configuration *b* or Case 2** The boundary conditions are,

1. inlet: Uniform Normal Velocity of Continuous type, $v_n = 0$.
2. outlet: Uniform Normal Velocity of Continuous type, $v_n = -1$.
3. The rest of the muffler is assumed to be rigid.

The boundary condition "Uniform Normal Velocity of Continuous type" implies that $v_n^+ = v_n^- = v_n$, where $+$ and $-$ correspond to the sides of the mesh in the same and opposite directions of the mesh normal.

## 4 Four-pole parameters for a muffler

The transfer matrix is

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} \quad (1)$$

where $p_1$ and $p_2$ are sound pressures at the inlet and outlet, and $v_1$ and $v_2$ are the particle velocities at the inlet and the outlet, respectively (refer to Figure 2); $T_{11}$, $T_{12}$, $T_{21}$, and $T_{22}$ are the four-pole parameters. The inlet and outlet points are chosen to be inside the inlet and outlet pipes close to the pipe ends.

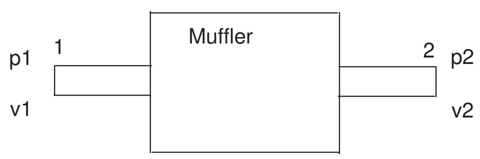


Figure 2: Four-poles.

To compute the tranfer matrix elements, also called four-pole parameters, we employ Two-source method [1]. In this method two different configurations of muffler are solved using Coustyx to obtain $p1$, $p2$, $v1$, and $v2$. Configuration $a$ or Case1 has the source or excitation at the inlet and a rigid outlet. The transfer matrix is

$$\begin{bmatrix} p_{1a} \\ v_{1a} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_{2a} \\ v_{2a} \end{bmatrix} \quad (2)$$

For Configuration $b$ or Case2 the source is switched to the side of the outlet and the inlet is made rigid. Therefore, for Configuration $b$ or Case2 the transfer relation is rewritten as

$$\begin{bmatrix} p_{2b} \\ -v_{2b} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}^{-1} \begin{bmatrix} p_{1b} \\ -v_{1b} \end{bmatrix}$$

or,

$$\begin{bmatrix} p_{2b} \\ v_{2b} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} T_{22} & T_{21} \\ T_{12} & T_{11} \end{bmatrix} \begin{bmatrix} p_{1b} \\ v_{1b} \end{bmatrix} \quad (3)$$

where $\Delta = T_{11}T_{22} - T_{12}T_{21}$, and the particle velocities $v_{1b}$ and $v_{2b}$ are in the direction of the flow for Configuration $b$, that is, from the outlet to the inlet.

The four-pole parameters are then solved in terms of pressure and velocities as follows

$$T_{11} = \frac{(p_{1a}v_{2b} + p_{1b}v_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{4}$$

$$T_{12} = \frac{(p_{1a}p_{2b} - p_{1b}p_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{5}$$

$$T_{21} = \frac{(v_{1a}v_{2b} - v_{1b}v_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{6}$$

$$T_{22} = \frac{(p_{2a}v_{1b} + v_{1a}p_{2b})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{7}$$

# 5 Transmission loss

The transmission loss for a muffler, in terms of four-pole parameters and inlet ($S_i$) and outlet ($S_o$) tube areas, is given by [2]

$$TL = 20\log_{10}\left[\frac{1}{2}\left|T_{11} + \frac{T_{12}}{Z_o} + T_{21}Z_o + T_{22}\right|\right] + 10\log_{10}\left(\frac{S_i}{S_o}\right) \tag{8}$$

For a simple expansion chamber, the transmission loss can be predicted by 1-dimensional plane-wave theory. The transmission loss using plane-wave solution is given by [3]

$$TL = 10\log_{10}\left\{1 + \frac{1}{4}\left(m - \frac{1}{m}\right)^2 \sin^2 kl_c\right\} \tag{9}$$

where $m = S_c/S_i$, $S_c$ is the area of cross-section of central chamber, and $S_i$ is the area of cross-section of the inlet pipe (here, $S_i = S_o$), and $l_c$ is the length of central chamber.

# 6 Results and validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx Indirect model. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. In the demo model, the analysis is performed for a frequency range of 50–3000 Hz with a frequency resolution of 50 Hz using the Fast Multipole Method (FMM) by running "Run Validation - FMM". Coustyx analysis results, along with the analytical solutions, are written to the output file "validation_results_fmm.txt". The results can be plotted using the matlab file "PlotResults.m".

The indirect BE model solves for the surface potentials $\mu$ and $\sigma$. These are, in turn, used to compute field point pressures and velocities at the two field points near the inlet ($p_1$, $v_1$) and the outlet ($p_2$, $v_2$). The field points are arbitrary selected to be 0.3 inches away from the inlet and outlet cross-sections within the muffler. The four-pole parameters are evaluated from the field point pressures and velocities computed from the Configurations $a$ (or Case 1) and $b$ (or Case 2).

Figure 3 shows comparisons of the transmission loss computed from Coustyx, 1-D plane wave theory and the measured data. The measured data is extracted from the experiment results published in [1]. The transmission loss derived from the plane-wave assumption defers vastly from Coustyx and experiments at higher frequencies due to the effects of higher modes. The plane-wave assumption is not valid at these frequencies. Coustyx results match well with the published measurements from experiments over the entire frequency range.

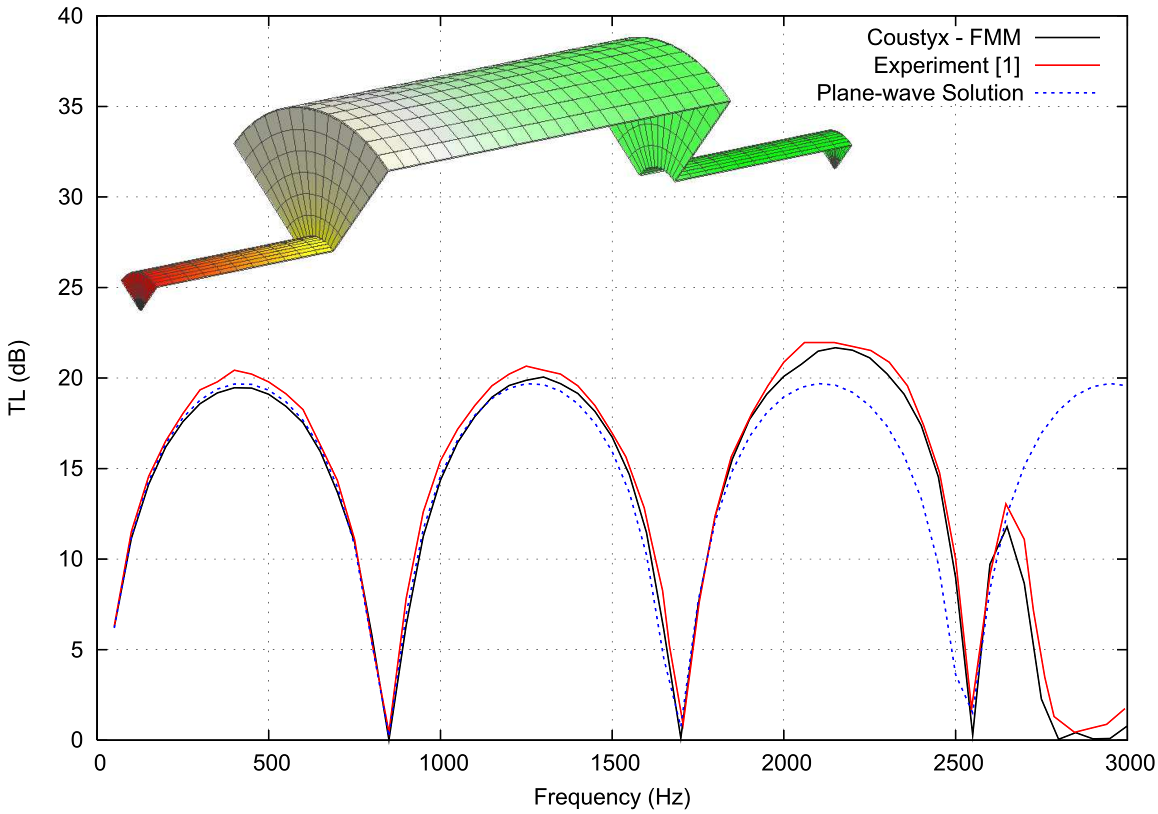


Figure 3: Transmission loss comparisons - Simple expansion chamber

# References


[1] Z. Tao and A.F. Seybert. A review of current techniques for measuring muffler transmission loss. *SAE International*, 2003.

[2] C. A. Brebbia and R. D. Ciskowski. *Boundary Element Methods in Acoustics.* Computational Mechanics Publications, 1991.

[3] E. B. Magrab. *Environmental Noise Control.* John Wiley & Sons, New York, 1975.