Muffler Transmission Loss – Simple Expansion Chamber

1 Introduction

The main objectives of this Demo Model are

- Demonstrate the ability of Coustyx to model a muffler using Indirect model and solve the acoustics problem to compute the transmission loss (TL) for the muffler.
- Compute TL using the following two methods:

  Four-pole Method: Four-pole parameters for the muffler are derived from Coustyx analysis and are used to compute TL.

  Three-point Method: The acoustic field pressure at three points are computed from Coustyx analysis and are used to compute TL.

- Validate Coustyx software by comparing TL computed from Coustyx to the analytical solution and published experimental measurements by Tao and Seybert [1].

2 Model description

In this example we model a simple expansion chamber and compute the transmission loss. The BEA solutions are compared with the experiment results extracted from the publication by Tao and Seybert [1].

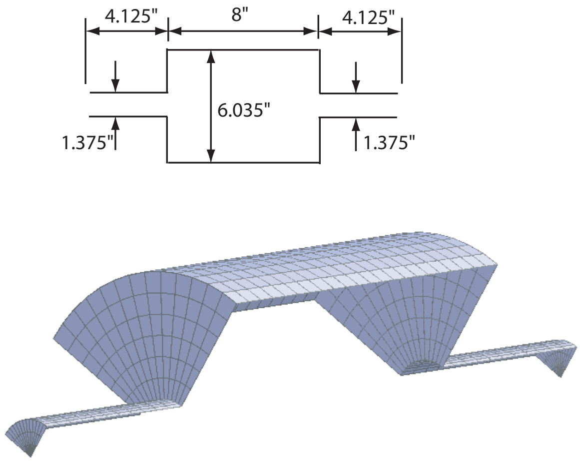

Figure 1: Boundary element mesh for the simple expansion chamber.

Only a quarter of the muffler is modeled to take advantage of the symmetry of the system and reduce the size of the problem. Figure 1 shows BEA mesh for the quarter model of the muffler. The dimensions of the muffler are given in inches. The fluid medium inside and around the muffler is air with sound speed $c = 13503.937\,\text{inch/s}(343\,\text{m/s})$ and mean density $\rho_o = 4.3714e-5\,\text{lb/inch}^3(1.21\,\text{kg/m}^3)$. Note that the units for the speed of sound and mean density are chosen to be consistent with the units for the length dimensions of the mesh. The characteristic impedance of air is $Z_o = \rho_o c$. The wavenumber at a frequency ω is $k = \omega/c$.

The BE mesh has linear coordinate connectivity as well as linear variable node connectivity. Coustyx indirect BE method is used to solve this problem.

3 Boundary Conditions

Different boundary conditions are applied for the two methods used to compute transmission loss.

3.1 Four-pole Method

Two BEM cases with different boundary conditions are run to compute the four-pole parameters.

Configuration *a* or Case 1 The boundary conditions are,

- inlet: Uniform Normal Velocity of Continuous type, $v_n = -1$ is defined to apply unit velocity in the direction of flow from the inlet to the outlet. Note: The normal velocity is defined with respect to the element normal. For the demo model, the element normals at the inlet are coming out of the muffler, which is in the opposite direction to the flow from the inlet to the outlet, hence a negative sign. The normal velocity is defined with respect to the element normal. At the inlet the element normals are all oriented outwards (coming out of the muffler), which is opposite to the direction of flow from the inlet to the outlet and hence a negative sign.
- outlet: Uniform Normal Velocity of Continuous type, $v_n = 0$.
- The rest of the muffler is assumed to be rigid.

Configuration *b* or Case 2 The boundary conditions are,

- inlet: Uniform Normal Velocity of Continuous type, $v_n = 0$.
- outlet: Uniform Normal Velocity of Continuous type, $v_n = -1$.
- The rest of the muffler is assumed to be rigid.

The boundary condition "Uniform Normal Velocity of Continuous type" implies that $v_n^+ = v_n^- = v_n$, where + and − correspond to the sides of the mesh on the positive and negative ends of a mesh normal.

3.2 Three-point Method

Only one BEM run is required to compute transmission loss from the three-point method. The boundary conditions employed in this method are,

- inlet: Uniform Normal Velocity of Continuous type, $v_n = -1$.
- outlet: The interior side of the muffler outlet is modeled to be anechoic. This boundary condition is applied as "Discontinuous" type with the following side boundary conditions:

  Side 1: This is the side on the positive end of a mesh normal. For the current mesh, Side 1 is on the exterior side of the outlet. Apply 'Dont Care' BC as we are not concerned with the external solution.

  Side 2: This is the side on the negative end of a mesh normal. For the current mesh, Side 2 is on the interior side of the outlet. To apply anechoic termination, select "Uniform Normal Velocity" BC. Enter a zero value for the structure normal velocity (v_{ns}^-) through 'Normal Velocity' and an 'Impedence' value equal to $\rho_o c$. That is impedance, $Z = \rho_o c = 0.59$. The anechoic termination BC is applied as, $\frac{p^-}{(v_n^- - v_{ns}^-)} = Z$, where p^- and v_n^- correspond to the pressure and particle normal velocity on the interior side of the outlet.

- The rest of the muffler is assumed to be rigid.

4 Transmission loss

Transmission loss for a muffler can be evaluated by a conventional four-pole method or by an efficient three-point method.
The four-pole method uses four-pole parameters to compute transmission loss for a muffler. These parameters are part of the transfer matrix connecting inlet and outlet pressures and velocities. We need two separate BEM runs to compute all the four-pole parameters. Hence, this method is slower than the three-point method, where only a single BEM run is required to evaluate muffler

transmission loss. However, the transfer matrix derived from the four-pole parameters could be used to represent the muffler in a system when multiple mufflers are connected with each other. On the other hand, the three-point method solves for the transmission loss only and nothing else.

4.1 Four-pole method

The transfer matrix in a four-pole method is given by

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} \tag{1}$$

where p_1 and p_2 are sound pressures at the inlet and outlet, and v_1 and v_2 are the particle velocities at the inlet and the outlet, respectively (refer to Figure 2); T_{11}, T_{12}, T_{21}, and T_{22} are the four-pole parameters. The inlet and outlet points are chosen to be inside the inlet and outlet pipes close to the pipe ends.

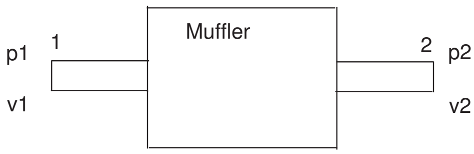

Figure 2: Four-poles.

To compute the transfer matrix elements, also called four-pole parameters, we employ Two-source method [1]. In this method two different configurations of muffler are solved using Coustyx to obtain $p1$, $p2$, $v1$, and $v2$. Configuration a or Case1 has the source or excitation at the inlet and a rigid outlet. The transfer matrix is

$$\begin{bmatrix} p_{1a} \\ v_{1a} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_{2a} \\ v_{2a} \end{bmatrix} \tag{2}$$

For Configuration b or Case2 the source is switched to the side of the outlet and the inlet is made rigid. Therefore, for Configuration b or Case2 the transfer relation is rewritten as

$$\begin{bmatrix} p_{2b} \\ -v_{2b} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}^{-1} \begin{bmatrix} p_{1b} \\ -v_{1b} \end{bmatrix}$$

or,

$$\begin{bmatrix} p_{2b} \\ v_{2b} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} T_{22} & T_{21} \\ T_{12} & T_{11} \end{bmatrix} \begin{bmatrix} p_{1b} \\ v_{1b} \end{bmatrix} \tag{3}$$

where $\Delta = T_{11}T_{22} - T_{12}T_{21}$, and the particle velocities v_{1b} and v_{2b} are in the direction of the flow for Configuration b, that is, from the outlet to the inlet.
The four-pole parameters are then solved in terms of pressure and velocities as follows

$$T_{11} = \frac{(p_{1a}v_{2b} + p_{1b}v_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{4}$$

$$T_{12} = \frac{(p_{1a}p_{2b} - p_{1b}p_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{5}$$

$$T_{21} = \frac{(v_{1a}v_{2b} - v_{1b}v_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{6}$$

$$T_{22} = \frac{(p_{2a}v_{1b} + v_{1a}p_{2b})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{7}$$

The transmission loss for a muffler, in terms of four-pole parameters and inlet (S_i) and outlet (S_o) tube areas, is given by [2]

$$TL = 20\log_{10}\left[\frac{1}{2}\left|T_{11} + \frac{T_{12}}{Z_o} + T_{21}Z_o + T_{22}\right|\right] + 10\log_{10}\left(\frac{S_i}{S_o}\right) \tag{8}$$

4.2 Three-point Method

In a three-point method transmission loss is evaluated from the field pressures measured at three points inside the muffler. Among the three points, two of them (points 1, and 2) are located in the inlet pipe and one (point 3) in the outlet pipe (refer to Figure 3). The two field points in the inlet pipe are used to extract the incoming wave pressure (p_i). The field point pressure at point 3 is the same as the transmitted wave pressure (p_t) in the outlet pipe, that is, $p_3 = p_t$. This is due to the specification of anechoic termination at the outlet, which by definition doesn't reflect waves back into the outlet pipe.

Due to the discontinuity in the impedance from the inlet pipe to the expansion chamber of the muffler, a portion of the incoming wave is reflected back to the source. Hence, pressures measured at points 1 and 2 in the inlet pipe are resultant of both the incoming (p_i) and reflected (p_r) waves and are given by [3],

$$p_1 = p_i \, e^{ikx_1} + p_r \, e^{-ikx_1} \tag{9}$$

$$p_2 = p_i \, e^{ikx_2} + p_r \, e^{-ikx_2} \tag{10}$$

where p_1, and p_2 are the pressure values; x_1, and x_2 are the locations of point 1 and point 2 respectively; $i = \sqrt{-1}$. Note that the above equations are little different from the equations specified in [3] due to the adoption of $e^{-i\omega t}$ convention in Coustyx, where ω is the angular frequency.

Solving the above two equations for p_i, we obtain

$$p_i = -\frac{1}{2i \sin k(x_2 - x_1)} \left[p_1 e^{-ikx_2} - p_2 e^{-ikx_1} \right] \tag{11}$$

where $\sin k(x_2 - x_1) \neq 0$ or $k(x_2 - x_1) \neq n\Pi$, $n = 0, 1, 2, ...$ Note that the spacing between the points 1 and 2 should be carefully chosen to satisfy this condition at all frequencies.

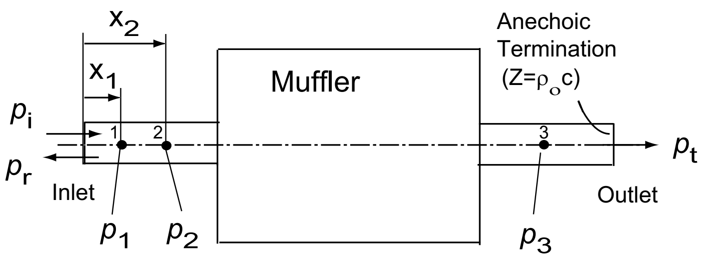

Figure 3: Three-point method [3].

The transmission loss for a muffler could be evaluated from the incoming (p_i) and the transmitted ($p_t = p_3$) wave pressures [3],

$$TL = 20 \log_{10} \left\{ \frac{|p_i|}{|p_3|} \right\} + 10 \log_{10} \left(\frac{S_i}{S_o} \right) \tag{12}$$

where S_i, and S_o are the inlet and outlet tube areas respectively.

5 Analytical Solution

For a simple expansion chamber, the transmission loss can be predicted by 1-dimensional plane-wave theory. The transmission loss using plane-wave solution is given by [4]

$$TL = 10 \log_{10} \left\{ 1 + \frac{1}{4} \left(m - \frac{1}{m} \right)^2 \sin^2 kl_c \right\} \tag{13}$$

where $m = S_c/S_i$, S_c is the area of cross-section of central chamber, and S_i is the area of cross-section of the inlet pipe (here, $S_i = S_o$), and l_c is the length of central chamber.

6 Results and validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx demo models. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. Coustyx analysis is performed for a frequency range of 50–3000 Hz with a frequency resolution of 50 Hz using the Fast Multipole Method (FMM) by running "Run Validation - FMM". Coustyx analysis results, along with the analytical solutions, are written to the output file "validation_results_fmm.txt". The results can be plotted using the matlab file "PlotResults.m".

6.1 Four-pole Method

Run the demo model "DemoModel-4PoleMethod" to evaluate muffler transmission loss by four-pole method. The indirect BE model solves for the surface potentials μ and σ. These are, in turn, used to compute field point pressures and velocities at the two field points near the inlet (p_1, v_1) and the outlet (p_2, v_2). The field points are arbitrary selected to be 0.3 inches away from the inlet and outlet cross-sections within the muffler. The four-pole parameters are evaluated from the field point pressures and velocities computed from the Configurations a (or Case 1) and b (or Case 2).
Figure 4 shows comparisons between the transmission loss derived from the four-pole parameters from Coustyx runs, 1-D plane wave theory and the measured data. The measured data is extracted from the experiment results published in [1]. The transmission loss derived from the plane-wave assumption defers vastly from Coustyx and experiments at higher frequencies due to the effects of higher modes. The plane-wave assumption is not valid at these frequencies. Coustyx results match well with the published measurements from experiments over the entire frequency range.

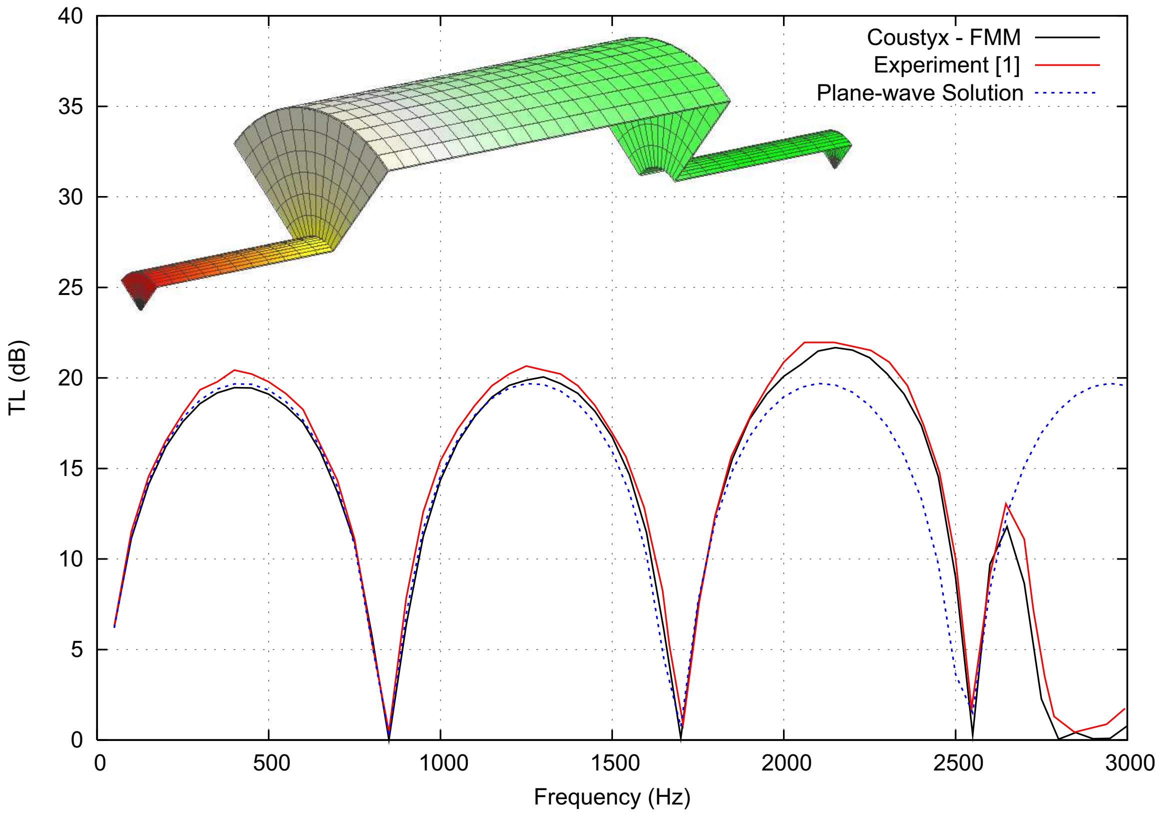

Figure 4: Transmission loss comparisons - Simple expansion chamber. Note that transmission loss for Coustyx-FMM case is computed from the four-pole method.

6.2 Three-point Method

Run the "DemoModel-3PointsMethod" to evaluate muffler transmission loss by three-point method. The indirect BE model solves for the surface potentials μ and σ. These are, in turn, used to compute field point pressures at the three field points at point 1 (p_1), point 2 (p_2) and the point 3 (p_3). The field point 1 is arbitrary selected to be 0.3 inches away from the inlet (that is, $x_1 = 0.3$), point 2 is 2.3 inches away from the inlet ($x_2 = 2.3$), and point 3 is at a distance of 0.3 inches from the outlet. The incoming wave pressure (p_i) and the transmitted wave pressure (p_3) are used to evaluate transmission loss. Figure 5 shows the transmission loss comparisons from Coustyx run, 1-D plane wave theory and the measured data.

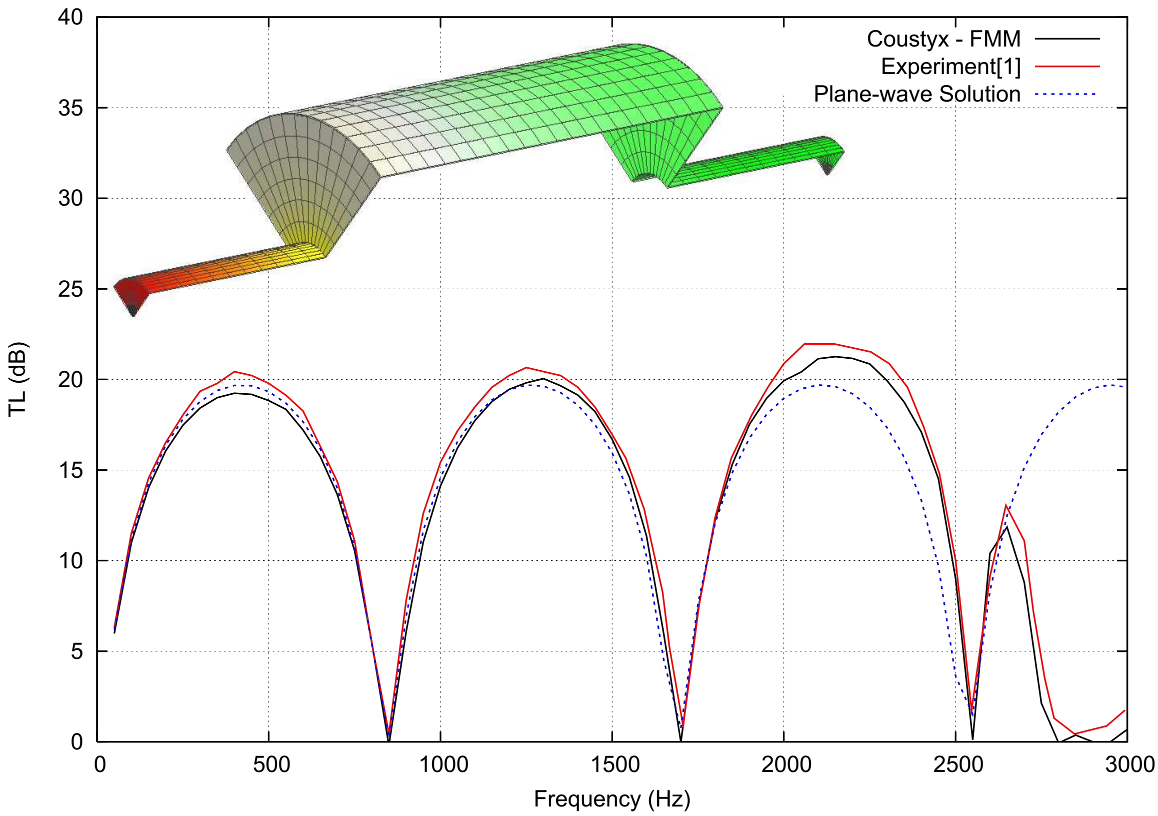

Figure 5: Transmission loss comparisons - Simple expansion chamber. Note that transmission loss for Coustyx-FMM case is computed from the three-point method.

References

[1] Z. Tao and A.F. Seybert. A review of current techniques for measuring muffler transmission loss. *SAE International*, 2003.

[2] C. A. Brebbia and R. D. Ciskowski. *Boundary Element Methods in Acoustics.* Computational Mechanics Publications, 1991.

[3] T.W. Wu and G. C. Wan. Muffler performance studies using a direct mixed-body boundary element method and a three-point method for evaluating transmission loss. *ASME Transaction, Journal of Vibration and Acoustics*, 118:479–484, 1996.

[4] E. B. Magrab. *Environmental Noise Control.* John Wiley & Sons, New York, 1975.