Transmission Loss - Perforated Muffler with a Flow Plug

1 Introduction

The main objectives of this Demo Model are

- Demonstrate the ability of Coustyx to model a perforated muffler using Indirect model and solve the acoustics problem to compute the Transmission Loss (TL) for the muffler.
- Compute TL using the following two methods:

  Four-pole Method: Four-pole parameters for the muffler are derived from Coustyx analysis and are used to compute TL.

  Three-point Method: The acoustic field pressure at three points are computed from Coustyx analysis and are used to compute TL.

- Validate Coustyx program by comparing the transmission loss computed from Coustyx to the experiment results from published material.

2 Model description

In this example we model a concentric-tube resonator with a flow-plug in the middle of a perforated tube and compute the transmission loss. The BEA solutions are compared with the experiment results extracted from the plot in Wu [1].

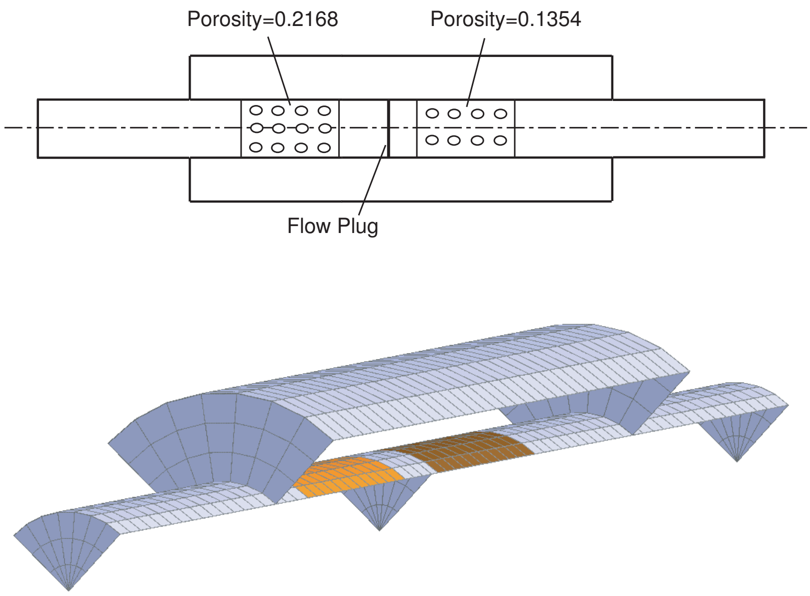

Figure 1: Boundary element mesh for the perforated muffler.

Only a quarter of the muffler is modeled to take advantage of the symmetry of the system and reduce the size of the problem. Figure 1 shows BEA mesh for the quarter model of the muffler. A flow-plug separates two perforated regions of porosities $\chi_1 = 0.2168$ and $\chi_2 = 0.1354$ (refer to Figure 1). Here, porosity is defined as the ratio of the open surface area to the total surface area of the perforated region. The dimensions of the muffler are in meters. The fluid medium inside

and around the muffler is air with sound speed $c = 343\,\mathrm{m/s}$ and mean density $\rho_o = 1.21\,\mathrm{kg/m^3}$. Note that the units for the speed of sound and mean density are chosen to be consistent with the units for the length dimensions of the mesh. The characteristic impedance of air is $Z_o = \rho_o c$. The wavenumber at a frequency ω is $k = \omega/c$.
The BE mesh has linear coordinate connectivity as well as linear variable node connectivity. Coustyx indirect BE method is used to solve this problem.

3 Boundary Conditions

Different boundary conditions are applied for the two methods used to compute transmission loss.

3.1 Four-pole Method

Two BEM cases with different boundary conditions are run to compute the four-pole parameters.

Configuration *a* or Case 1 The boundary conditions are,

1. inlet: Uniform Normal Velocity of Continuous type, $v_n = 1$.
2. outlet: Uniform Normal Velocity of Continuous type, $v_n = 0$.
3. perforated region 1: Uniform Perforated, Porosity $\chi_1 = 0.2168$.
4. perforated region 2: Uniform Perforated, Porosity $\chi_2 = 0.1354$.
5. The rest of the muffler is assumed to be rigid.

Configuration *b* or Case 2 The boundary conditions are,

1. inlet: Uniform Normal Velocity of Continuous type, $v_n = 0$.
2. outlet: Uniform Normal Velocity of Continuous type, $v_n = 1$.
3. perforated region 1: Uniform Perforated, Porosity $\chi_1 = 0.2168$.
4. perforated region 2: Uniform Perforated, Porosity $\chi_2 = 0.1354$.
5. The rest of the muffler is assumed to be rigid.

The boundary condition "Uniform Normal Velocity of Continuous type" implies that $v_n^+ = v_n^- = v_n$, where $+$ and $-$ correspond to the sides of the mesh on the positive and negative ends of a mesh normal.

3.2 Three-point Method

Only one BEM run is required to compute transmission loss from the three-point method. The boundary conditions employed in this method are,

- inlet: Uniform Normal Velocity of Continuous type, $v_n = 1$.
- outlet: The interior side of the muffler outlet is modeled to be anechoic. This boundary condition is applied as "Discontinuous" type with the following side boundary conditions:

  Side 1: This is the side on the positive end of a mesh normal. For the current mesh, Side 1 is on the interior side of the outlet. To apply anechoic termination, select "Uniform Normal Velocity" BC. Enter a zero value for the structure normal velocity (v_{ns}^+) through 'Normal Velocity' and an 'Impedence' value equal to $\rho_o c$. That is impedance, $Z = \rho_o c = 415.03$. The anechoic termination BC is applied as, $\frac{p^+}{(v_n^+ - v_{ns}^+)} = Z$, where p^+ and v_n^+ correspond to the pressure and particle normal velocity on the interior side of the outlet.

  Side 2: This is the side on the negative end of a mesh normal. For the current mesh, Side 2 is on the exterior side of the outlet. Apply 'Dont Care' BC as we are not concerned with the external solution.

- perforated region 1: Uniform Perforated, Porosity $\chi_1 = 0.2168$.
- perforated region 2: Uniform Perforated, Porosity $\chi_2 = 0.1354$.
- The rest of the muffler is assumed to be rigid.

Note that a perforated boundary condition defines a special type of transfer relation between the pressure, normal velocity on either side of the surface. The transfer relation used in Coustyx relates the pressure and velocity with the frequency (f), and hole density or porosity (χ), and is given by (refer to Sullivan and Crocker [2])

$$p^+ - p^- = -\rho_o c\zeta\left[v_n - \bar{v}_{sn}\right]$$

where,

$$\zeta = \frac{\left[5.783 \times 10^{-3} - i(4.819 \times 10^{-5})f\right]}{\chi}$$

is the non-dimensional transfer impedance of the perforated surface derived by normalizing the empirical formula (for ζ) given by Sullivan and Crocker [2] with the characteristic impedance of the medium; p^+ and p^- are surface pressures on side 1 and side 2 respectively; v_n is the acoustic normal velocity, and v_{sn} is the specified structure normal velocity.

4 Transmission loss

Transmission loss for a muffler can be evaluated by a conventional four-pole method or by an efficient three-point method.
The four-pole method uses four-pole parameters to compute transmission loss for a muffler. These parameters are part of the transfer matrix connecting inlet and outlet pressures and velocities. We need two separate BEM runs to compute all the four-pole parameters. Hence, this method is slower than the three-point method, where only a single BEM run is required to evaluate muffler transmission loss. However, the transfer matrix derived from the four-pole parameters could be used to represent the muffler in a system when multiple mufflers are connected with each other. On the other hand, the three-point method solves for the transmission loss only and nothing else.

4.1 Four-pole method

The transfer matrix in a four-pole method is given by

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} \tag{1}$$

where p_1 and p_2 are sound pressures at the inlet and outlet, and v_1 and v_2 are the particle velocities at the inlet and the outlet, respectively (refer to Figure 2); T_{11}, T_{12}, T_{21}, and T_{22} are the four-pole parameters. The inlet and outlet points are chosen to be inside the inlet and outlet pipes close to the pipe ends.

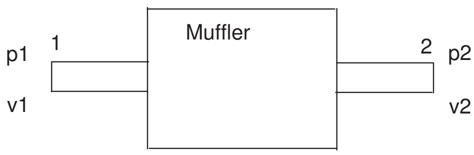

Figure 2: Four-poles.

To compute the transfer matrix elements, also called four-pole parameters, we employ Two-source method [3]. In this method two different configurations of muffler are solved using Coustyx to obtain $p1$, $p2$, $v1$, and $v2$. Configuration a or Case1 has the source or excitation at the inlet and a rigid outlet. The transfer matrix is

$$\begin{bmatrix} p_{1a} \\ v_{1a} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_{2a} \\ v_{2a} \end{bmatrix} \tag{2}$$

For Configuration b or Case2 the source is switched to the side of the outlet and the inlet is made rigid. Therefore, for Configuration b or Case2 the transfer relation is rewritten as

$$\begin{bmatrix} p_{2b} \\ -v_{2b} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}^{-1} \begin{bmatrix} p_{1b} \\ -v_{1b} \end{bmatrix}$$

or,

$$\begin{bmatrix} p_{2b} \\ v_{2b} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} T_{22} & T_{21} \\ T_{12} & T_{11} \end{bmatrix} \begin{bmatrix} p_{1b} \\ v_{1b} \end{bmatrix} \tag{3}$$

where $\Delta = T_{11}T_{22} - T_{12}T_{21}$, and the particle velocities v_{1b} and v_{2b} are in the direction of the flow for Configuration b, that is, from the outlet to the inlet.
The four-pole parameters are then solved in terms of pressure and velocities as follows

$$T_{11} = \frac{(p_{1a}v_{2b} + p_{1b}v_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{4}$$

$$T_{12} = \frac{(p_{1a}p_{2b} - p_{1b}p_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{5}$$

$$T_{21} = \frac{(v_{1a}v_{2b} - v_{1b}v_{2a})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{6}$$

$$T_{22} = \frac{(p_{2a}v_{1b} + v_{1a}p_{2b})}{(p_{2a}v_{2b} + p_{2b}v_{2a})} \tag{7}$$

The transmission loss for a muffler, in terms of four-pole parameters and inlet (S_i) and outlet (S_o) tube areas, is given by [4]

$$TL = 20\log_{10}\left[\frac{1}{2}\left|T_{11} + \frac{T_{12}}{Z_o} + T_{21}Z_o + T_{22}\right|\right] + 10\log_{10}\left(\frac{S_i}{S_o}\right) \tag{8}$$

4.2 Three-point Method

In a three-point method transmission loss is evaluated from the field pressures measured at three points inside the muffler. Among the three points, two of them (points 1, and 2) are located in the inlet pipe and one (point 3) in the outlet pipe (refer to Figure 3). The two field points in the inlet pipe are used to extract the incoming wave pressure (p_i). The field point pressure at point 3 is the same as the transmitted wave pressure (p_t) in the outlet pipe, that is, $p_3 = p_t$. This is due to the specification of anechoic termination at the outlet, which by definition doesn't reflect waves back into the outlet pipe.
Due to the discontinuity in the impedance from the inlet pipe to the expansion chamber of the muffler, a portion of the incoming wave is reflected back to the source. Hence, pressures measured at points 1 and 2 in the inlet pipe are resultant of both the incoming (p_i) and reflected (p_r) waves and are given by [5],

$$p_1 = p_i\, e^{ikx_1} + p_r\, e^{-ikx_1} \tag{9}$$

$$p_2 = p_i\, e^{ikx_2} + p_r\, e^{-ikx_2} \tag{10}$$

where p_1, and p_2 are the pressure values; x_1, and x_2 are the locations of point 1 and point 2 respectively; $i = \sqrt{-1}$. Note that the above equations are little different from the equations specified in [5] due to the adoption of $e^{-i\omega t}$ convention in Coustyx, where ω is the angular frequency.
Solving the above two equations for p_i, we obtain

$$p_i = -\frac{1}{2i\sin k(x_2 - x_1)}\left[p_1e^{-ikx_2} - p_2e^{-ikx_1}\right] \tag{11}$$

where $\sin k(x_2 - x_1) \neq 0$ or $k(x_2 - x_1) \neq n\Pi$, $n = 0, 1, 2, ...$ Note that the spacing between the points 1 and 2 should be carefully chosen to satisfy this condition at all frequencies.

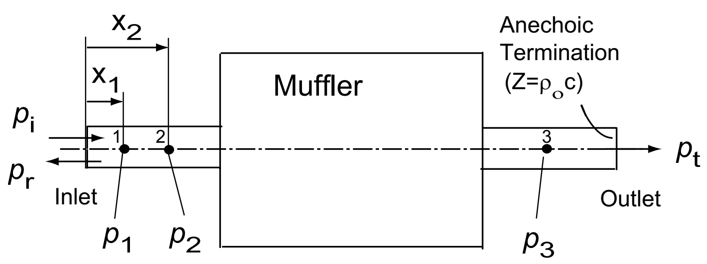

Figure 3: Three-point method [5].

The transmission loss for a muffler could be evaluated from the incoming (p_i) and the transmitted ($p_t = p_3$) wave pressures [5],

$$TL = 20\log_{10}\left\{\frac{|p_i|}{|p_3|}\right\} + 10\log_{10}\left(\frac{S_i}{S_o}\right) \tag{12}$$

where S_i, and S_o are the inlet and outlet tube areas respectively.

5 Results and validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx demo models. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. Coustyx analysis is performed for a frequency range of 50–3250 Hz with a frequency resolution of 50 Hz using the Fast Multipole Method (FMM) by running "Run Validation - FMM". Coustyx analysis results, along with the analytical solutions, are written to the output file "validation_results_fmm.txt". The results can be plotted using the matlab file "PlotResults.m".

5.1 Four-pole Method

Run the demo model "DemoModel-4PoleMethod" to evaluate muffler transmission loss by four-pole method. The indirect BE model solves for the surface potentials μ and σ. These are, in turn, used to compute field point pressures and velocities at the two field points near the inlet (p_1, v_1) and the outlet (p_2, v_2). The field points are arbitrary selected to be 0.01 meters away from the inlet and outlet cross-sections within the muffler. The four-pole parameters are evaluated from the field point pressures and velocities computed from the Configurations a (or Case 1) and b (or Case 2).
Figure 4 shows comparisons of the transmission loss computed from Coustyx, and the measured data from experiments. The measured data is extracted from the plot in Wu [1]. Coustyx results match fairly well with the published measurements over the entire frequency range.

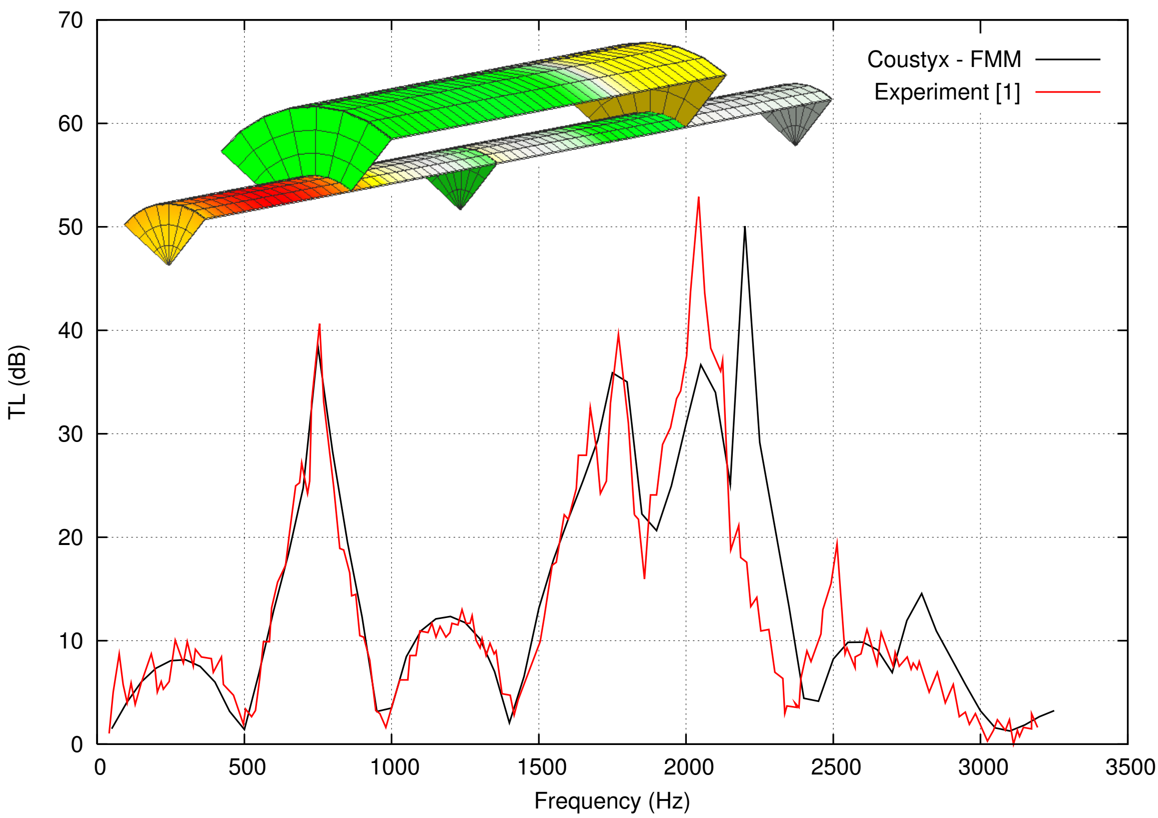

Figure 4: Transmission loss comparisons - Perforated muffler with a flow-plug. Note that transmission loss for Coustyx-FMM case is computed from the four-pole method.

5.2 Three-point Method

Run the demo model "DemoModel-3PointsMethod" to evaluate muffler transmission loss by three-point method. The indirect BE model solves for the surface potentials μ and σ. These are, in turn, used to compute field point pressures at the three field points at point 1 (p_1), point 2 (p_2) and the point 3 (p_3). The field point 1 is arbitrary selected to be 0.05 meters away from the inlet (that is, $x_1 = 0.05$), point 2 is 0.1 meters away from the inlet ($x_2 = 1.0$), and point 3 is at a distance of 0.05 meters from the outlet. The incoming wave pressure (p_i) and the transmitted wave pressure (p_3) are used to evaluate transmission loss. Figure 5 shows the transmission loss comparisons from Coustyx, and the measured data extracted from [1].

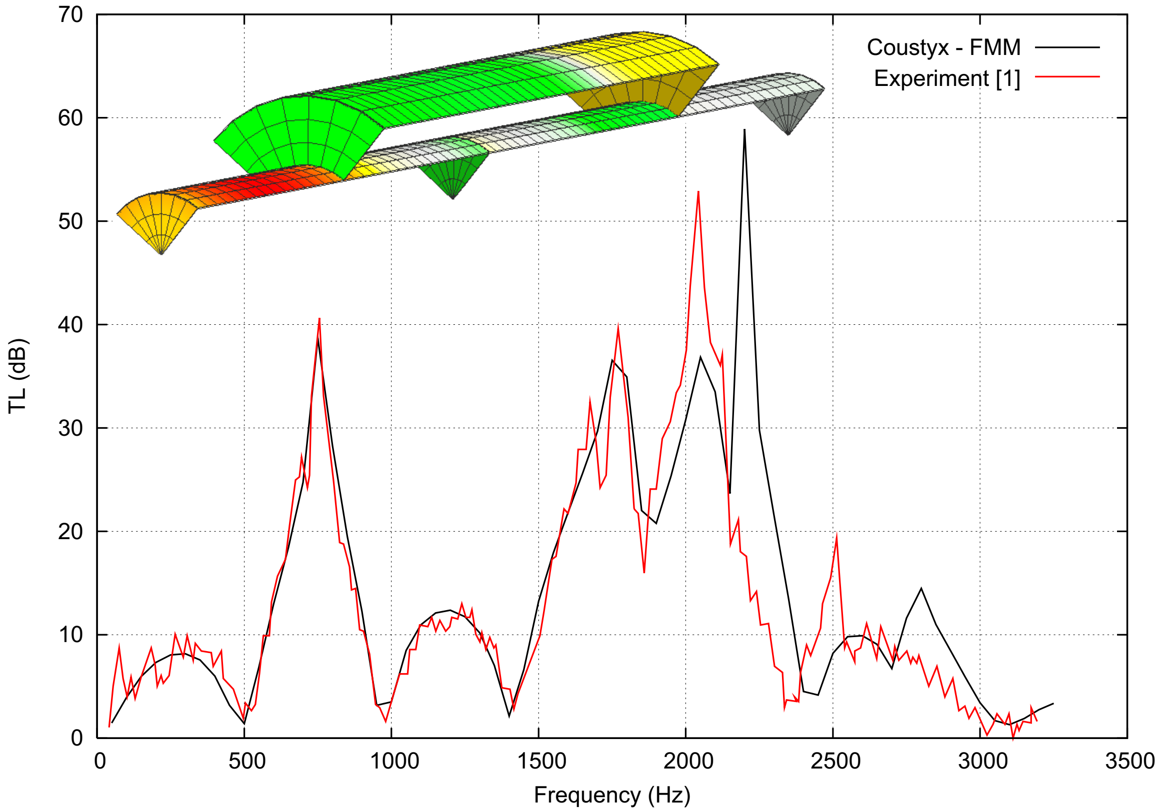

Figure 5: Transmission loss comparisons - Perforated muffler with a flow-plug. Note that transmission loss for Coustyx-FMM case is computed from the three-point method.

References

[1] T.W. Wu, editor. *Boundary Element Acoustics - Fundamentals and Computer Codes.* WIT Press, Southampton, Boston, 2000.

[2] J. W. Sullivan and M. J. Crocker. Analysis of concentric-tube resonators having unpartitioned cavities. *Journal of Acoustic Society of America*, 64:207–215, 1978.

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[5] T.W. Wu and G. C. Wan. Muffler performance studies using a direct mixed-body boundary element method and a three-point method for evaluating transmission loss. *ASME Transaction, Journal of Vibration and Acoustics*, 118:479–484, 1996.