Reciprocity in Acoustics

1 Introduction

The main objective of this Demo Model is to

- Validate Coustyx program by verifying the *reciprocity principle* in acoustics in the presence of a plate barrier.

This example answers the problem statement in Pierce [1]:

> A barrier extending to some height h is to be erected between a noise source and a region where quiet is desired. One side of the barrier is to be treated with special sound-absorbing material; the other side is to be left untreated. On which side should the treatment be applied?

2 Model description

We model a plate of size 1 m x 1 m to represent a barrier between the source and observation point (refer to Figure 1). The fluid medium surrounding the plate is air with sound speed $c = 343\,\text{m/s}$ and mean density of air $\rho_o = 1.21\,\text{kg/m}^3$. The characteristic impedance of air $Z_o = \rho_o c = 415.03\,\text{kg/m}^2\text{/s}$. The wavenumber at a frequency ω is given as $k = \omega/c$. The mesh normal (n) is in the positive z direction. A monopole source of unit volume velocity is defined at a distance 1 *m* away from the center, perpendicular to the plate, at (0.5,0.5,-1), where the center of plate is at (0.5,0.5,0). The field point pressure is measured at an observation point 1 *m* away on the opposite side of the plate, that is at (0.5,0.5,1). One side of the plate is treated with a special material of impedance $Z = 1.5\rho_o c - i8\rho_o c$ and the other side is modeled as rigid.

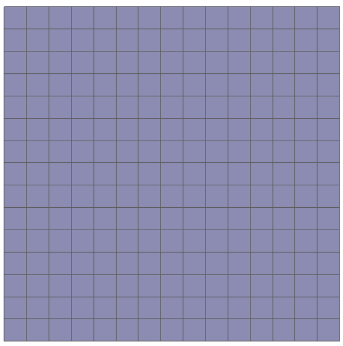

Figure 1: Boundary element mesh a plate.

3 Boundary conditions

To identify the side of the plate to be treated for effective noise reduction, we consider two different boundary condition (BC) configurations. Both of them are implemented through applying "Discontinuous" type BC on the elements of the plate.

Case 1 The observer side of the plate is treated with sound-absorbing material of impedance Z (Figure 3). The side boundary conditions are

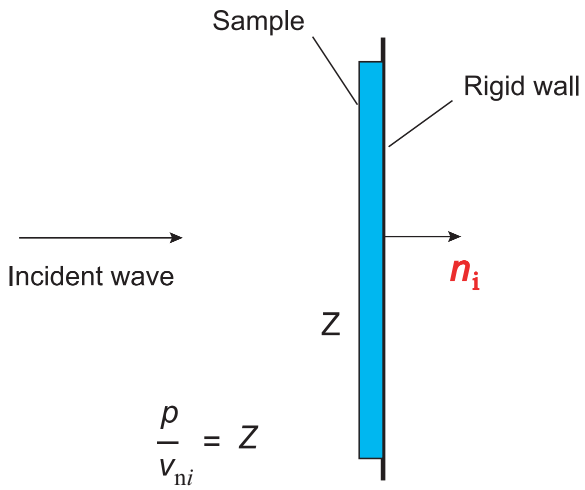

Figure 2: Definition of impedance. Note p is the pressure and v_{ni} is the particle normal velocity in n_i direction at the surface of the material.

side 1 : This is the side of the plate on the positive side of the mesh normal n (or on observer side).

A *uniform normal velocity* of amplitude zero ($v_{n\_wall} = 0$), and *impedance* $Z = 1.5\rho_o c - i8\rho_o c$ is applied on this side. The impedance (Z) relates the pressure (p) at the surface of the acoustic material to the particle normal velocity (v_{ni}) at the surface as

$$\frac{p}{v_{ni} - v_{n\_wall}} = Z \tag{1}$$

The particle normal velocity v_{ni} at any point on the sample surface is the *inward* (pointing into the surface and out of the fluid) normal component of the fluid velocity at that point. Figure 2 shows the impedance definition followed in Coustyx.

side 2 : This is the side of the plate on the negative side of the mesh normal n (or on source side).

Uniform Normal Velocity, $v_n = 0$.

Case 2 The source side of the plate is treated with sound-absorbing material of impedance Z (Figure 4). The side boundary conditions are

side 1 : This is the side of the plate on the positive side of the mesh normal n (or on observer side).

Uniform Normal Velocity, $v_n = 0$.

side 2 : This is the side of the plate on the negative side of the mesh normal n (or on source side).

A *uniform normal velocity* of amplitude zero ($v_{n\_wall} = 0$), and *impedance* $Z = 1.5\rho_o c - i8\rho_o c$ is applied on this side. The impedance (Z) relates the pressure (p) at the surface of the acoustic material to the particle normal velocity (v_{ni}) as in Equation 1 (Figure 2).

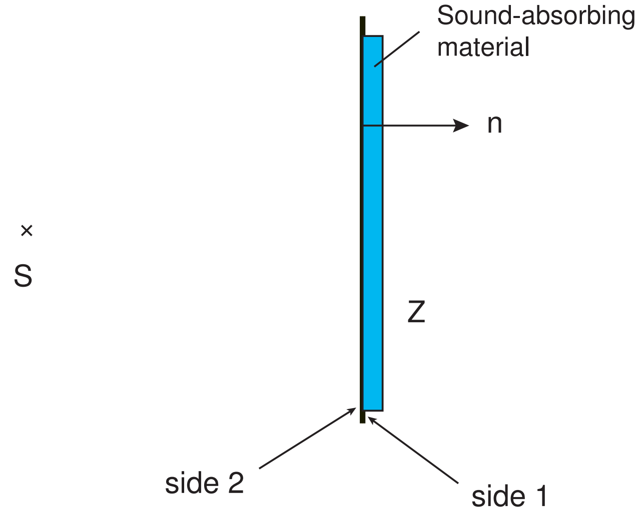

Figure 3: Case 1: Plate treated with sound-absorbing material of impedance Z on the observer side. Note S–source, O–observation point, n–element normal.

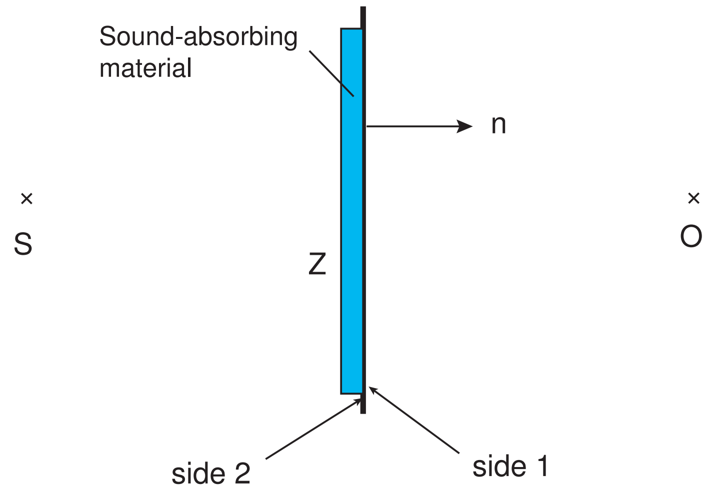

Figure 4: Case 2: Plate treated with sound-absorbing material of impedance Z on the source side. Note S–source, O–observation point, n–element normal.

4 Results and validation

Figure 3 shows *Case 1* with the observer side of the plate treated with a sound-absorbing material. Coustyx indirect BE model for *Case 1* is run using analysis sequence defined in the model. The analysis sequence stores the analysis parameters such as frequency of analysis, solution methods to be used, etc., required to do an analysis. In the demo model, the analysis is performed at a frequency of 500 Hz using the Fast Multipole Method (FMM).
The field point pressure at the observation point at (0.5,0.5,1) for *Case 1* is (184.459,78.616).
Figure 4 shows *Case 2* with the source side of the plate treated with a sound-absorbing material. Coustyx indirect BE model for *Case 2* is run using analysis sequence defined in the model. The analysis is performed at a frequency of 500 Hz using the Fast Multipole Method (FMM).
The field point pressure at the observation point at (0.5,0.5,1) for *Case 2* is (184.452, 78.749).
The pressure at the observation point remains the same from both the cases ($error < 0.07\%$). Thus, we conclude that the Coustyx solution remains the same irrespective of the side of the plate treated with sound-absorbing material.

4.1 Reciprocity principle

The above conclusion can also be arrived through the application of *principle of reciprocity* in acoustics. From [1]:

> Reciprocity refers to situations for which a magnitude associated with an "effect" at a point is unchanged when the locations of "cause" and "point of observation" are interchanged.

As the source and observation points are symmetrically located on opposite sides of the plate, by the application of reciprocity, we can conclude that it makes no difference which side is treated with sound-absorbing material [1].

References

[1] A. D. Pierce. *Acoustics - An introduction to its physical principles and applications*. Acoustical Society of America, 1991. Pages 195–199.