Transmission Loss due to a Movable Wall

In this example problem, we study the transmission loss between two domains separated by wall. The wall is modeled as a movable mass connected to ground by a lumped spring and damper. To the left of the wall is the source domain, where an acoustic pressure wave is incident on the wall. The incident pressure wave is reflected by the wall back into the source domain. The incident wave also sets the wall in vibratory motion and causes a transmitted wave in the receiver domain. We compute the transmission loss by the wall using Coustyx, and compare with analytical solutions, in different frequency regimes. Analytical solution is available when we consider the special case of a linear duct with a walled partition.

The source domain is a linear duct, 6 in in diameter and 2 ft long. The left wall of source domain is vibrating with a velocity of 3.407 mm/s which provides the excitation to the system. This velocity amplitude is that of an acoustic plane wave with a pressure amplitude of $\sqrt{2}$ Pa which corresponds to Sound Pressure Level of 94 dB, (back calculated from a plane wave of 1 Pa rms pressure, just to use some reasonable numbers). The velocity amplitude of such a wave will be $\frac{\sqrt{2}}{\rho_0 c} = 3.407\,\mathrm{mm/s}$. The common wall between the source and the receiver domains has a mass of 0.01 Kg and is connected to the ground with a lumped stiffness of $k = 6.32E4\,\mathrm{N/m}$ and lumped damping $c = 0.503\,\mathrm{Ns/m}$. This structure has an in-vacuo mode at 400.1 Hz and modal damping ratio of $\zeta = 0.01$.

The receiver domain is also a linear duct of the same diameter (6 in) and length (2 ft). On the right wall of the receiver domain, an anechoic termination boundary condition is specified, so the transmitted wave travels forward without reflection from the terminating boundary. This is achieved by using an impedance boundary condition at the right wall with a specific impedance of $Z = P/V = \rho_0 c$ to match the characteristic impedance of the medium.

In this article, we describe the steps in preparing the model, and compare the numerical solution from Coustyx with the exact analytical solution. The derivation of the analytical solution makes use of the concept of structure mobility.

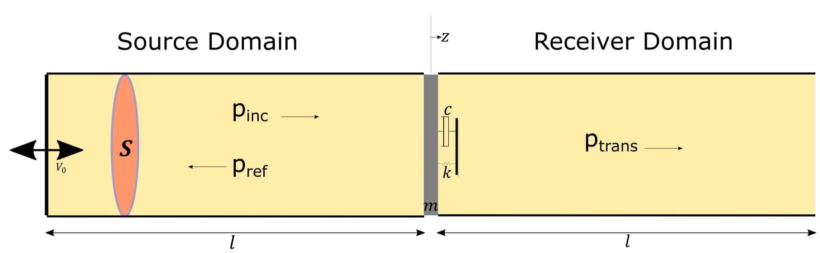

Figure 1: Movable wall partition separating two fluid domains.

1 Model Preparation

The source domain is a duct of length $l = 0.6096\,\mathrm{m}$ and cross sectional area of $S = 1.824E-2\,\mathrm{m}^2$. The BEM model is created by importing the surface mesh of the duct, and specifying the fluid domain to be bounded and on the interior of the mesh. The side of the surface mesh on the which the fluid domain of interest lies, is obtained by first inspecting the orientation of the surface BEM mesh. If the element normals are pointing to the outside, we specify that the fluid domain is on the *negative* side of the mesh, as we are interested in the interior problem. The BE Mesh for the receiver domain is also created by importing the corresponding surface mesh.

The structure model corresponding the wall (named `wall`), along with its rigid body mode at 400.10 Hz are imported from a Nastran `op2` file. The coupling type of the structure is set as *Coupled* as we have to include the two way interaction of this structure with both the fluid domains at $z = 0$.

The next task is to apply the correct boundary conditions to various faces of the BEM mesh. On the left face of the `SourceDomain` mesh `SourceSideBEMesh`, we specify a *uniform normal velocity boundary* of $v_n = -3.407E-3$ m/s. This corresponds to $v_z = 3.407E-3$ m/s, as the domain normal is oriented in the negative z direction. The normals used in the boundary condition definitions in MultiDomain problems are always the domain normals and not the mesh normals. Zero normal velocity (rigid boundary condition) is specified on the side walls. On the right face at $z = 0$, a structure velocity boundary condition named `wallbc` is specified. The details of this boundary condition are shown in Figure 2. This boundary condition is also be applied to the left wall of the `ReceiverDomain`

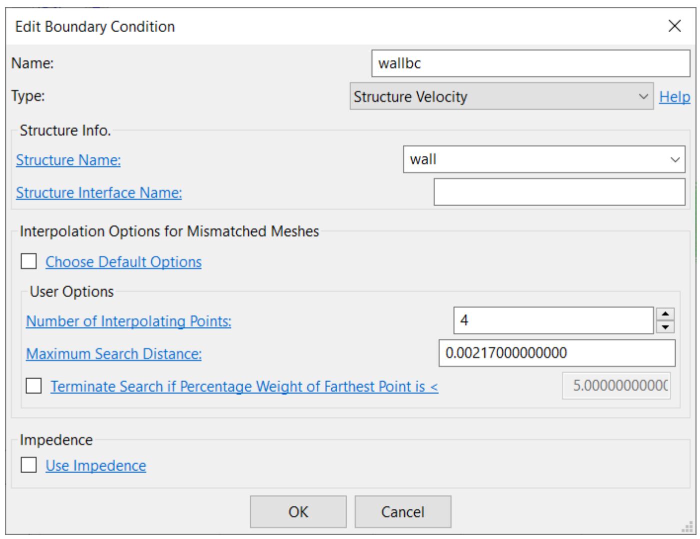

Figure 2: Structure velocity boundary condition applied to the right surface of the source domain and the left surface of the receiver domain at $z = 0$.

2 Analytical Solution

2.1 Driving point mobility of the Single Degree of Freedom System

We assume time harmonic variation of $e^{-i\omega t}$ where ω is the excitation frequency. Let $f(t) = Fe^{-i\omega t}$ be the applied force and $u(t) = Ue^{-i\omega t}$ be the deflection of the mass m in the z-direction.

$$m\ddot{u} + c\dot{u} + k = f \tag{1}$$

$$(-m\omega^2 - ic\omega + k)U = F \tag{2}$$

$$\frac{U}{F} = \frac{1}{(-m\omega^2 - ic\omega + k)} \tag{3}$$

The driving point mobility $Y = V_m/F = -i\omega U/F$ is given as

$$\begin{aligned} Y = \frac{V_m}{F} &= \frac{-i\omega}{(-m\omega^2 - ic\omega + k)} \\ Y(\omega) &= \frac{-ir}{(m\omega_n)((1-r^2) - i(2\zeta r))} \end{aligned} \tag{4}$$

where $r = \omega/\omega_n$ is the ratio of the excitation frequency ω to the natural frequency ω_n of the SDOF system. In the SDOF system used in this example, mass $m = 0.01\,\text{Kg}$, stiffness $k = 6.32 \times 10^4\text{N/m}$ and damping $c = 0.503\text{N s/m}$. This corresponds to an in vacuo natural frequency of 400.1 Hz and damping ratio of $\zeta = 0.01$.

2.2 Acoustic field inside the source and receiver domain

We can assume the sound field in the source domain to be of the form

$$\begin{aligned} p_{source}(z) &= p_{inc} + p_{ref} \qquad (5) \\ &= Ae^{ikz} + Be^{-ikz} \qquad (6) \end{aligned}$$

The expression for velocity in the z-direction is

$$v_{source}(z) = \frac{A}{Z_0}e^{ikz} - \frac{B}{Z_0}e^{-ikz} \qquad (7)$$

One equation for the coefficients A and B are obtained from the prescribed velocity boundary condition at the left end $z = -l$. The velocity at $z = -l$ must be $v_{source}(0) = V_0$. This yields

$$\begin{aligned} V_0 &= \frac{A}{Z_0}e^{-ikl} - \frac{B}{Z_0}e^{ikl} \\ V_0 Z_0 &= Ae^{-ikl} - Be^{ikl} \qquad (8) \end{aligned}$$

The sound field in the receiver domain will be forward propagating wave as it has anechoic termination at the right end.

$$p_{trans}(z) = Ce^{ikz} \qquad (9)$$

As the mass moves, the air in the source domain and the receiver domain moves with it (velocity continuity in the normal direction between the structure and the fluid). Therefore, at $z = 0$, the v_z of the fluid in the source domain is equal to velocity of the mass, which in turn is equal to the v_z of the fluid in the receiver domain. This yields,

$$\begin{aligned} \frac{A}{Z_0} - \frac{B}{Z_0} &= V_m = \frac{C}{Z_0} \\ A - B &= V_m Z_0 = C \qquad (10) \end{aligned}$$

The velocity of the mass V_m is obtained using the driving point mobility $Y(\omega)$ as

$$V_m = \{p_{inc}(0) + p_{ref}(0) - p_{trans}(0)\}SY(\omega) \qquad (11)$$

where S is the cross-sectional area over which the sound pressure loading acts on the structure. Equation 11 yields

$$\begin{aligned} \{p_{inc}(0) + p_{ref}(0) - p_{trans}(0)\}SY(\omega) &= \frac{C}{Z_0} \\ \{A + B - C\}SY(\omega)Z_0 &= C \qquad (12) \end{aligned}$$

Using Equation 8, Equation 10 and Equation 12, the system of equations governing the pressure wave amplitudes can be written as:

$$\begin{bmatrix} e^{-ikl} & -e^{ikl} & 0 \\ 1 & -1 & -1 \\ SY(\omega)Z_0 & SY(\omega)Z_0 & -SY(\omega)Z_0 - 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} V_0 Z_0 \\ 0 \\ 0 \end{Bmatrix} \qquad (13)$$

Using the multilinear property of determinants, the determinant Δ of the coefficient matrix in Equation 13 can be simplified as

$$\begin{aligned}
\Delta &= \begin{vmatrix} e^{-ikl} & -e^{ikl} & 0 \\ 1 & -1 & -1 \\ SY(\omega)Z_0 & SY(\omega)Z_0 & -SY(\omega)Z_0 - 1 \end{vmatrix} \\
&= \begin{vmatrix} e^{-ikl} & -e^{ikl} & 0 \\ 1 & -1 & -1 \\ SY(\omega)Z_0 & SY(\omega)Z_0 & -SY(\omega)Z_0 \end{vmatrix} + \begin{vmatrix} e^{-ikl} & -e^{ikl} & 0 \\ 1 & -1 & -1 \\ 0 & 0 & -1 \end{vmatrix} \\
&= SY(\omega)Z_0 \begin{vmatrix} e^{-ikl} & -e^{ikl} & 0 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} + \begin{vmatrix} e^{-ikl} & -e^{ikl} & 0 \\ 1 & -1 & -1 \\ 0 & 0 & -1 \end{vmatrix} \\
&= SY(\omega)Z_0[e^{-ikl}(1+1) + e^{ikl}(-1+1)] - 1(-e^{-ikl} + e^{ikl}) \\
&= 2SY(\omega)Z_0 e^{-ikl} + e^{-ikl} - e^{ikl} \\
&= [2SY(\omega)Z_0 + 1]e^{-ikl} - e^{ikl}
\end{aligned} \tag{14}$$

Using Cramer's rule, the pressure amplitudes A, B and C can be determined as

$$\begin{aligned}
A &= \frac{1}{\Delta} \begin{vmatrix} V_0 Z_0 & -e^{ikl} & 0 \\ 0 & -1 & -1 \\ 0 & SY(\omega)Z_0 & -SY(\omega)Z_0 - 1 \end{vmatrix} \\
&= \frac{[2SY(\omega)Z_0 + 1]V_0 Z_0}{\Delta}
\end{aligned} \tag{15}$$

$$\begin{aligned}
B &= \frac{1}{\Delta} \begin{vmatrix} e^{-ikl} & V_0 Z_0 & 0 \\ 1 & 0 & -1 \\ SY(\omega)Z_0 & 0 & -SY(\omega)Z_0 - 1 \end{vmatrix} \\
&= \frac{V_0 Z_0}{\Delta}
\end{aligned} \tag{16}$$

$$\begin{aligned}
C &= \frac{1}{\Delta} \begin{vmatrix} e^{-ikl} & -e^{ikl} & V_0 Z_0 \\ 1 & -1 & 0 \\ SY(\omega)Z_0 & SY(\omega)Z_0 & 0 \end{vmatrix} \\
&= \frac{[2SY(\omega)Z_0]V_0 Z_0}{\Delta}
\end{aligned} \tag{17}$$

From Equation 15, Equation 16 and Equation 17, the pressure Reflection Ratio $R = B/A$ is given as

$$R = \frac{B}{A} = \frac{1}{2SY(\omega)Z_0 + 1} \tag{18}$$

The pressure Transmission Ratio $T = C/A$ is given as

$$T = \frac{C}{A} = \frac{2SY(\omega)Z_0}{2SY(\omega)Z_0 + 1} \tag{19}$$

It is seen that for this problem $R + T = 1$. The Transmission Loss due to the wall is given as

$$\begin{aligned}
TL &= -10\log_{10}|T|^2 \\
&= -10\log_{10}|\frac{2SY(\omega)Z_0}{2SY(\omega)Z_0 + 1}|^2 \\
&= -10\log_{10}|\frac{1}{1 + \frac{1}{2SY(\omega)Z_0}}|^2
\end{aligned} \tag{20}$$

From Equation 20, when $2SY(\omega)Z_0 >> 1$, which happens when excitation frequency ω is close to the natural frequency ω_n, the transmission loss due to the wall will be close to 0 dB. The wall will be effective barrier when $2SY(\omega)Z_0 << 1$. Since the cross-sectional area of the duct S and the characteristic impedance of the fluid medium Z_0 are constant, what effectively governs the transmission loss of a barrier wall is its driving point mobility. Intuitively, if the wall has a lower mobility, it will have smaller velocity amplitude for the same applied force and will transmit less.

In Figure 3, we are plotting the absolute value of $2SY(\omega)Z_0$ as a function of frequency. Also plotted on the graph are the stiffness line (mobility if the wall is massless) and the mass line (assuming zero stiffness). It is clear that at low frequencies, the wall mobility is stiffness controlled (red and black lines coincide) and at very high frequencies the wall mobility is mass controlled (red and green lines coincide). Near the resonance frequency, between 207.36 Hz and 538.31 Hz, and we do not expect much transmission loss, as the system mobility is very high ($2SY(\omega)Z_0 >> 1$). At lower frequencies, wall stiffness contributes more to the observed TL, while the mass effect is dominant at higher frequencies. To give good TL, an ideal wall will have low natural frequency and high mass.

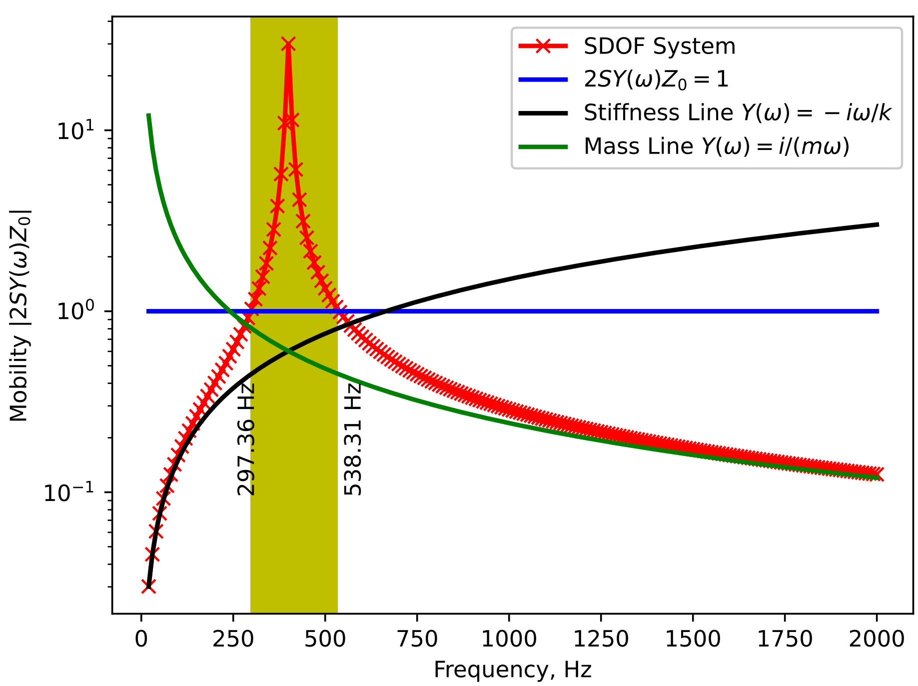

Figure 3: Wall mobility as a function of frequency.

2.3 Comparison with Exact Solution

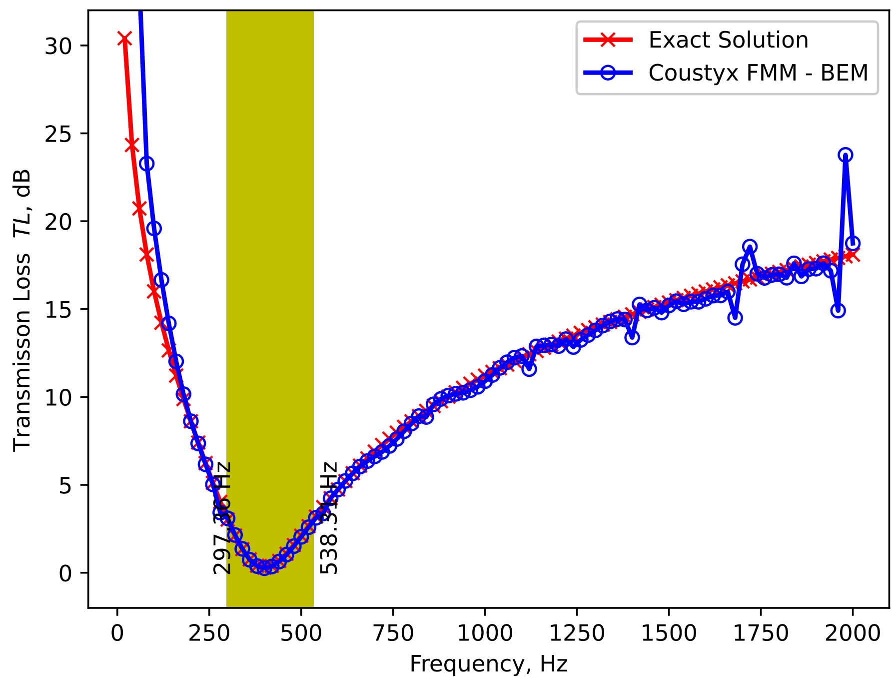

Figure 4: Wall mobility as a function of frequency.

We ran the analysis sequence `ComputeTL - FMM` to compute the Transmission Loss between 20 Hz to 1000 Hz in 20 Hz increments. When computing the numerical solution using Coustyx, it is not possible to compute the pressure amplitudes A, B in the source domain explicitly. Instead, we extract the total surface pressures at the left and right end of the source domain and use those to resolve the amplitudes of the incident and the reflected waves. For computing the transmitted wave amplitude C, it is sufficient to obtain the surface pressure from the left end of the receiver domain.

The transmission loss values obtained from exact solution and Coustyx are plotted in Figure 4. As seen from Figure 4, excellent agreement is seen between the Coustyx solution and the exact solution. As expected, close to the natural frequency when the wall mobility is high, the Transmission Loss is quite low at less than 3 dB. At the duct system natural frequencies, the coefficient matrix is singular and its determinant Δ is zero. The transmission ratio when using the exact solution is smooth as the Δ which divides the numerator and denominator terms cancels out cleanly and does not enter the calculation. However, in numerical data from Coustyx, this is not possible, hence a little jumpiness is seen at frequencies corresponding the to duct system natural frequencies, otherwise excellent agreement is seen between Coustyx and the exact solutions.