

One Dimensional Waves in a Cube with Anechoic Termination

1 Introduction

The main objectives of this Demo Model are to

- Demonstrate the ability of Coustyx to model a cube interior (bounded) problem using a MultiDomain model.
- Derive analytical solutions for the interior (bounded) problem of the cube based on one dimensional wave equation.
- Validate Coustyx software by comparing the Coustyx results to the analytical solutions.

2 Model description

We model a cube of size $1\text{ m} \times 1\text{ m} \times 1\text{ m}$. The fluid medium inside the cube is air with sound speed $c = 343\text{ m/s}$ and mean density $\rho_o = 1.21\text{ kg/m}^3$. The characteristic impedance of air $Z_o = \rho_o c = 415.03\text{ Rayl}$. The wavenumber at a frequency ω is given as $k = \omega/c$. All the faces of the cube are rigid except the faces at $x = 0$ and $x = 1$. On the face at $x = 0$ an excitation of constant pressure $\tilde{p} = P_0 = 1\text{ Pa}$ is applied, and the face at $x = 1$ is treated with anechoic termination. The BE mesh of the cube is shown in Figure 1.

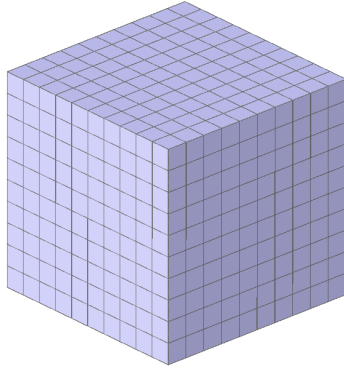


Figure 1: Boundary element mesh for a cube.

3 Boundary Conditions

The cube face at $x = 0$ is applied with constant pressure, that is $\tilde{p} = 1\text{ Pa}$. In Coustyx, this is applied using the boundary condition type “Uniform Pressure” with unit amplitude.

The cube face at $x = 1$ is applied with anechoic termination, that is, at any point on the surface the pressure and normal velocity are related by $\frac{\tilde{p}}{\tilde{v}_n} = \rho_o c$, where \tilde{v}_n is the normal velocity pointing away from the domain. This boundary condition is applied in Coustyx using the type “Arbitrary Uniform” with $\alpha = 1$, $\beta = -\rho_o c$, $\gamma = 0$, where α, β, γ are from $\alpha\tilde{p} + \beta\tilde{v}_n = \gamma$.

All the other faces are applied with zero normal velocity using the boundary condition type “Uniform Normal Velocity” (rigid boundary condition).

4 Analytical solution

The solution to a general 1-D wave equation is of the form

$$\tilde{p}(x) = Ae^{ikx} + Be^{-ikx} \quad (1)$$

The boundary conditions on the cube faces at $x = 0$ and $x = 1$ are,

$$\begin{aligned} \tilde{p} &= P_0, x = 0 \\ \frac{\tilde{p}}{\tilde{v}_n} &= \frac{\tilde{p}}{\tilde{v}_x} = \rho_o c, x = 1 \end{aligned} \quad (2)$$

The normal n on the cube face at $x = 1$ points away from the interior domain in $+x$ direction. Hence, at $x = 1$, $\tilde{v}_x = \tilde{v}_n$, where v_x is the x component of particle velocity.

The pressure at any point inside the cube is given by

$$\tilde{p}(x) = P_0 e^{ikx} \quad (3)$$

The velocity in $+x$ direction is

$$\tilde{v}_x = \frac{P_0}{\rho_o c} e^{ikx} \quad (4)$$

5 Results and validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx MultiDomain model. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. In this Demo Model, the analysis is performed at a frequency $f = 54.59Hz$ using the Fast Multipole Method (FMM) by running "Run Validation - FMM". Coustyx analysis results, along with the analytical solutions, are written to the output file "validation_results_fmm.txt". The results can be plotted using the matlab file "PlotResults.m".

Coustyx uses Direct BE method to solve the acoustic problem. In Direct BE method, the primary variables are the pressure and the pressure gradient on the boundary. Field point solutions are then computed from the surface solutions.

Figure 2 shows field point pressure variation with x at $(x, 0.5, 0.5)$ from both Coustyx and Analytical methods. The comparisons show very good agreement between the two solutions.

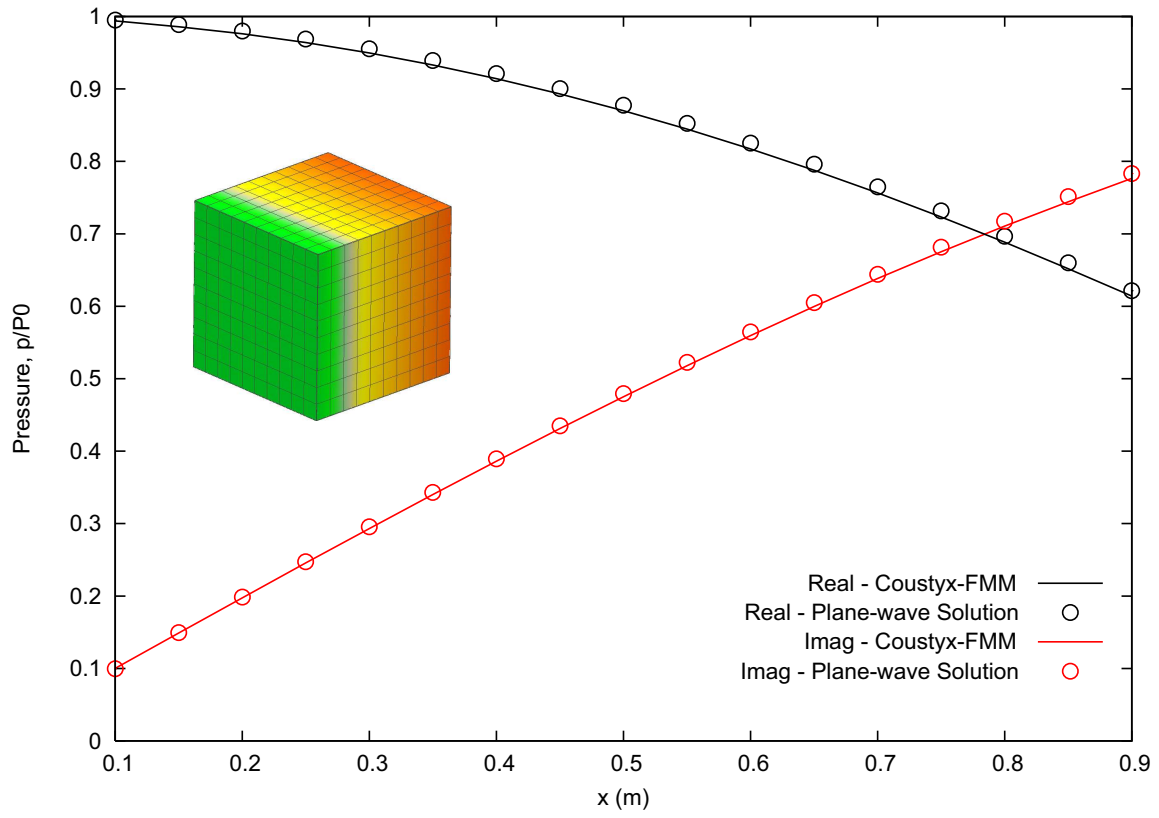


Figure 2: Field pressure comparisons at $(x,0.5,0.5)$ from Coustyx and analytical methods.