One Dimensional Waves in a Cube

1 Introduction

The main objectives of this Demo Model are to

- Demonstrate the ability of Coustyx to model a cube interior (bounded) problem using a MultiDomain model.
- Derive analytical solutions for the interior (bounded) problem of the cube based on one dimensional wave equation.
- Validate Coustyx software by comparing the Coustyx results to the analytical solutions.

2 Model description

We model a cube of size $1\,\mathrm{m} \times 1\,\mathrm{m} \times 1\,\mathrm{m}$. The fluid medium inside the cube is air with sound speed $c = 343\,\mathrm{m/s}$ and mean density $\rho_o = 1.21\,\mathrm{kg/m^3}$. The characteristic impedance of air $Z_o = \rho_o c = 415.03\,Rayl$. The wavenumber at a frequency ω is given as $k = \omega/c$. All the faces of the cube are rigid except the face at $x = 0$, which is vibrating with unit normal velocity in $+x$ direction. The BE mesh of the cube is shown in Figure 1.

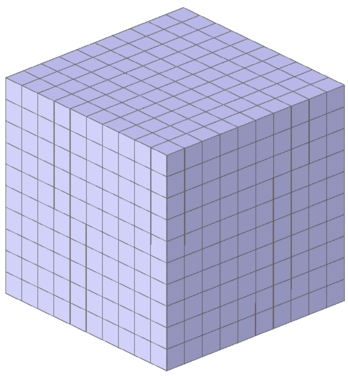

Figure 1: Boundary element mesh for a cube.

3 Boundary Conditions

The cube face at $x = 0$ is vibrating with unit velocity in $+x$ direction, that is $\tilde{v}_x = 1\,\mathrm{m/s}$. In Coustyx, this boundary condition is applied as an "Uniform Normal Velocity", $v_n = -\tilde{v}_x$; where v_n is the normal velocity pointing away from the cube interior. Note that all boundary conditions in a MultiDomain model are defined with respect to the *Domain Normal*, which always points away from the domain of interest. For the cube interior problem, the interior domain is the domain of interest; hence, domain normal is pointing away from the interior domain, that is, pointing away from the face of the cube at $x = 0$. All the other faces are applied with zero normal velocity (rigid boundary condition).

4 Analytical solution

The cube interior problem is analytically solved using 1-D plane wave assumptions.
The solution to a general 1-D wave equation is of the form

$$\tilde{p}(x) = Ae^{ikx} + Be^{-ikx} \tag{1}$$

Assume the following general boundary conditions (with the rest of the faces of the cube assumed to be rigid)

$$\begin{aligned} \tilde{v}_x &= v_{xo}, x = 0 \\ \tilde{v}_x &= v_{xl}, x = l \end{aligned} \tag{2}$$

The pressure at any point in the plane at x is given by

$$\tilde{p}(x) = \frac{i\rho c(v_{xo} \cos k(l-x) - v_{xl} \cos kx)}{\sin kl} \tag{3}$$

The velocity in $+x$ direction is

$$\tilde{v}_x = \frac{v_{xo} \sin k(l-x) + v_{xl} \sin kx}{\sin kl} \tag{4}$$

5 Results and validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx MultiDomain model. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. In this Demo Model, the analysis is performed at a frequency $f = 54.59Hz$ using the Fast Multipole Method (FMM) by running “Run Validation - FMM”. Coustyx analysis results, along with the analytical solutions, are written to the output file “validation_results_fmm.txt”. The results can be plotted using the matlab file “PlotResults.m”.
Coustyx uses Direct BE method to solve the acoustic problem. In Direct BE method, the primary variables are the pressure and the pressure gradient on the boundary. Field point solutions are then computed from the surface solutions.
Figure 2 shows field point pressure variation with x at (x,0.5,0.5) from both Coustyx and Analytical methods. The comparisons show very good agreement between the two solutions.

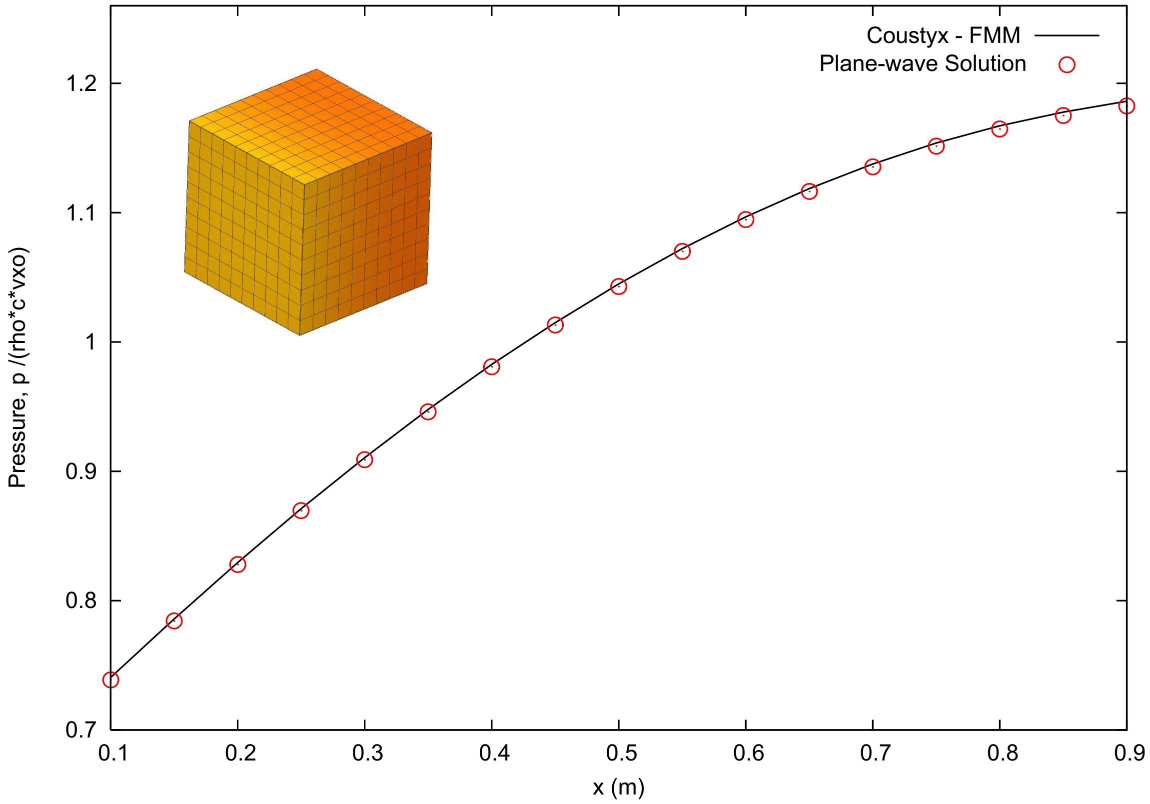

Figure 2: Field pressure comparisons at (x,0.5,0.5) from Coustyx and analytical methods.