Acoustic Field inside an Enclosed Room with a Point Source

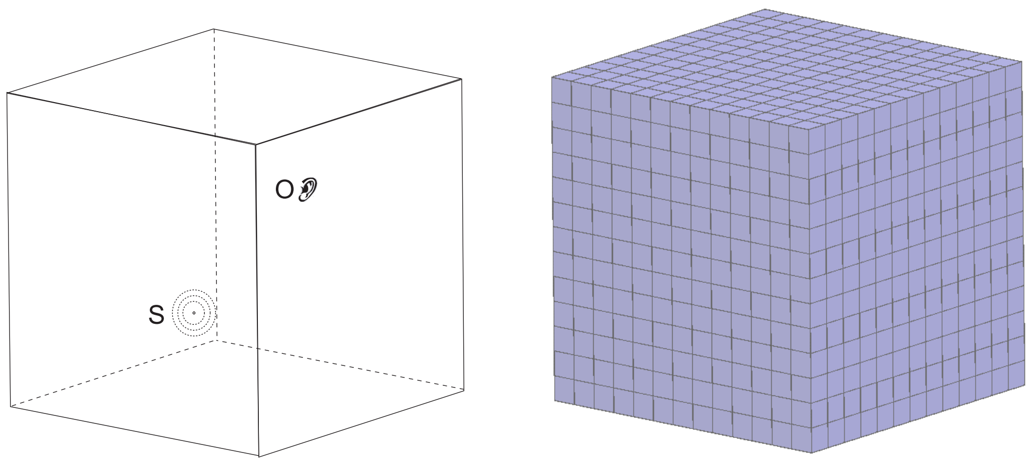
1 Introduction

The main objectives of this Demo Model are to

- Demonstrate the ability of Coustyx to model an enclosed room with a point source using Coustyx MultiDomain model and solve for the acoustic field distribution inside the room.
- Derive analytical solution using the modal theory of room acoustics.
- Validate Coustyx software by comparing the results from Coustyx to the analytical solutions in the presence of acoustic sources.

2 Model description

We model the room to be a cube of size $1\,\text{m} \times 1\,\text{m} \times 1\,\text{m}$. The fluid medium inside the cube is air with mean density $\rho_o = 1.21\,\text{kg/m}^3$ and sound speed $c = 343 - i * 10\,\text{m/s}$. A complex speed of sound introduces damping in the system. The imaginary part of the speed of sound should always be negative for a decaying sound wave. The wavenumber at a frequency ω is given as $k = omega/c$. A monopole source of unit volume velocity is introduced at (0.1,0.2,0.3) to simulate the point source in the room. All the faces of the cube are assumed to be rigid. The BE mesh of the cube is shown in Figure 1.



(a) Acoustic problem.

(b) Boundary element mesh

Figure 1: Cubic room with a point source. Note S–source, O–observation point

3 Boundary conditions

In the Coustyx MultiDomain model, the rigid wall condition is simulated by applying the boundary conditions on the cube faces as “Uniform Normal Velocity” type with zero amplitudes. That is, $v_n = 0$, where v_n is the particle normal velocity on the face of the cube in the *Domain Normal* direction. Note that all boundary conditions in a MultiDomain model are defined with respect to the *Domain Normal*, which always points away from the domain of interest. For this example, the interior domain is the domain of interest; hence, domain normal is pointing away from the cube interior.

4 Analytical solution

We first compute modes for the room (of size $L_x \times L_y \times L_z$) with the rigid walls boundary condition. These modes are then used in modal expansion to evaluate field point pressure at any point inside the room.

4.1 Eigenvalue problem

Table 1: Natural frequencies of a rigid cube

Frequency (Hz)	n_x	n_y	n_z
0	0	0	0
171.5	0	0	1
171.5	0	1	0
171.5	1	0	0
242.5	0	1	1
242.5	1	0	1
242.5	1	1	0
297.1	1	1	1
343	0	0	2
343	0	2	0
343	2	0	0
383.5	0	1	2
383.5	0	2	1
383.5	1	0	2
383.5	1	2	0
383.5	2	0	1
383.5	2	1	0
420.1	1	1	2
420.1	1	2	1
420.1	2	1	1
485.1	0	2	2
485.1	2	0	2
485.1	2	2	0

The eigen-function $\Psi(\mathbf{x}, n)$ satisfies the Helmholtz equation at any point inside the cube [1]

$$\left[\nabla^2 + k_n^2\right] \Psi(\mathbf{x}, n) = 0 \tag{1}$$

where k_n^2 is the eigenvalue.
The eigen-function should also satisfy the rigid boundary conditions on the faces of the cube

$$\frac{\partial \Psi(\mathbf{x}, n)}{\partial \hat{\mathbf{n}}} = 0$$

where $\hat{\mathbf{n}}$ is the surface normal.
To solve the eigenvalue problem, we assume that the eigen-function can be factored into a form $\Psi(\mathbf{x}, n) = X(x)Y(y)Z(z)$. The Helmholtz equation is reduced to

$$\frac{1}{X(x)} \frac{\partial^2 X(x)}{\partial x^2} + \frac{1}{Y(y)} \frac{\partial^2 Y(y)}{\partial y^2} + \frac{1}{Z(z)} \frac{\partial^2 Z(z)}{\partial z^2} + k_n^2 = 0$$

Applying separation of variables, the independent equation in terms of the variable x is written as

$$\frac{\partial^2 X(x)}{\partial x^2} + k_x^2 X(x) = 0$$

The function $X(x)$ also satisfies the rigid boundary conditions at $x = 0$ and $x = L_x$, that is,

$$\frac{\partial X}{\partial x} = 0$$

Solving for $X(x)$ in the above equations, we obtain

$$\begin{array}{l} X(x) = \cos(k_x x) \\ k_x = \frac{n_x \pi}{L_x}, n_x = 0, 1, 2, \ldots \end{array} \tag{2}$$

Applying similar conditions to $Y(y)$ and $Z(z)$, the eigen-function is derived.

$$\Psi(\mathbf{x}, n_x, n_y, n_z) = \cos\left(\frac{n_x \pi}{L_x} x\right) \cos\left(\frac{n_y \pi}{L_y} y\right) \cos\left(\frac{n_z \pi}{L_z} z\right) \tag{3}$$

The eigenvalue $k_n^2 = k_x^2 + k_y^2 + k_z^2$ is

$$k_n^2 = \left(\frac{n_x \pi}{L_x}\right)^2 + \left(\frac{n_y \pi}{L_y}\right)^2 + \left(\frac{n_z \pi}{L_z}\right)^2 \tag{4}$$

where $k_n = \omega/c = 2\pi f/c$, f is the frequency in Hz. Table 4.1 shows the list of eigenvalues lying between 0–500 Hz.

4.2 Modal expansion

The acoustic field pressure p at any point $\mathbf{x}$ (inside the cube) due to the presence of a point source at $\mathbf{x}_s$ satisfies the Helmholtz equation

$$\nabla^2 p + k^2 p = \varpi \delta(\mathbf{x} - \mathbf{x}_s) \tag{5}$$

where the source strength $\varpi = ik\rho c\beta_o$, β_o is the volume velocity; $\delta(\mathbf{x} - \mathbf{x}_s)$ is the Dirac delta function.
The modal eigen-functions derived above form a complete set. Hence, the acoustic solution p inside the room can be approximated as a linear combination of these eigen-functions.

$$p = \sum_n A_n \Psi(\mathbf{x}, n) \tag{6}$$

A_n is the mode participation coefficient.
We compute the mode participation coefficient A_n by substituting Equation 6 into Equation 5, that is,

$$\sum_n A_n \left[k^2 - k_n^2\right] \Psi(\mathbf{x}, n) = \varpi \delta(\mathbf{x} - \mathbf{x}_s) \tag{7}$$

Multiply Equation 7 with $\Psi(\mathbf{x}, n')$ and integrate over the entire volume. Using the orthogonality of eigen-functions, and the properties of Dirac delta function, the coefficient A_n is derived.

$$A_n = q_n \frac{\varpi}{V\left[k^2 - k_n^2\right]} \cos\left(\frac{n_x \pi}{L_x} x_s\right) \cos\left(\frac{n_y \pi}{L_y} y_s\right) \cos\left(\frac{n_z \pi}{L_z} z_s\right) \tag{8}$$

where $V = L_x L_y L_z$ is the volume of the cube; $q_n = q_x(n_x) q_y(n_y) q_z(n_z)$, $q_x(n_x) = 1$ for $n_x = 0$, and $q_x(n_x) = 2$ for $n_x \neq 0$, similarly for q_y and q_z.

5 Results and validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx MultiDomain model. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. In the demo model, the analysis is performed for a frequency range of 50–500 Hz with a frequency resolution of 10 Hz using the Fast Multipole Method (FMM) by running “Run Validation - FMM”. Coustyx analysis results, along with the analytical solutions, are written to the output file “validation_results_fmm.txt”. The results can be plotted using the matlab file “PlotResults.m”.
Coustyx MultiDomain model uses Direct BE method to solve the acoustic problem. In Direct BE method, the primary variables are the pressure and the pressure gradient on the boundary. Field point pressure at an interior observation point (0.7,0.7,0.7) is computed from both Coustyx and analytical methods and are compared in Figure 2 for all frequencies. The comparisons show very good agreement between these two solutions. Figure 2 also shows the resonance peaks at the natural frequencies of the rigid cube (Table 4.1).

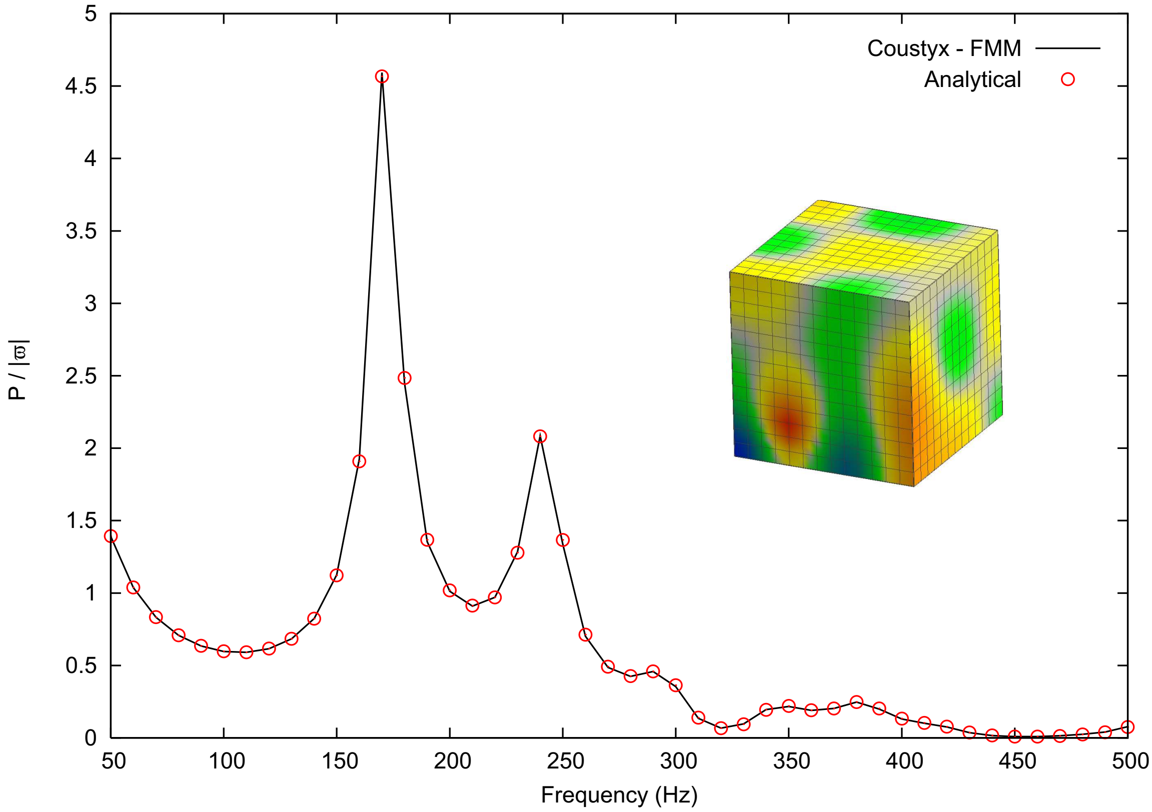


Figure 2: Field pressure comparisons at (0.7,0.7,0.7) inside a rigid cube with a monopole source from Coustyx and analytical methods. Note that P is the field point pressure and $\varpi = ik\rho_o c\beta_o$, where β_o is the volume velocity of the monopole source.

References

[1] A. D. Pierce. *Acoustics - An Introduction to Its Physical Principles and Applications*. Acoustical Society of America, 1991. Page 284.