Pulsating Sphere Radiation Problem

1 Introduction

The main objectives of this Demo Model are

- Demonstrate the ability of Coustyx to model a pulsating sphere radiation problem using a MultiDomain model.
- Derive analytical solution for the exterior (unbounded) problem of the pulsating sphere.
- Validate Coustyx software by comparing Coustyx results to analytical solutions.

2 Model description

We model a sphere of radius $a = 1\,\mathrm{m}$. The fluid medium surrounding the sphere is air with sound speed $c = 343\,\mathrm{m/s}$ and mean density $\rho_o = 1.21\,\mathrm{kg/m^3}$. The characteristic impedance of air $Z_o = \rho_o c = 415.03\,Rayl$. The wavenumber at a frequency ω is given as $k = \omega/c$. The sphere is pulsating with a uniform radial velocity $v_r = 1\,\mathrm{m/s}$. The BE mesh of the sphere is shown in Figure 1. Only an octant of the sphere is modeled to exploit the symmetry in the problem.

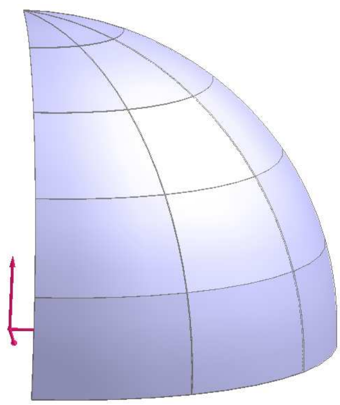

Figure 1: Boundary element mesh for an octant model of a sphere.

3 Boundary Conditions

The pulsating sphere is vibrating with a uniform outward radial velocity, $vr = 1$. In Coustyx, this boundary condition is applied as an "Uniform Normal Velocity", $v_n = -v_r$; where v_n is the normal velocity pointing into the sphere. Note that all boundary conditions in a MultiDomain model are defined with respect to the *Domain Normal*, which always points away from the domain of interest. For the sphere radiation problem, the exterior domain is the domain of interest; hence, domain normal is pointing away from the exterior domain, that is, pointing into the sphere.

4 Analytical solution

The pulsating sphere with a uniform radial velocity has exact solution. For the exterior problem, the pressure at a distance r from the center of the sphere is given by

$$\tilde{p}(r) = \frac{a}{r} v_r Z_o \frac{-ika}{1-ika} \exp\left(ik(r-a)\right) \tag{1}$$

$$\left(\frac{\partial \tilde{p}}{\partial r}\right)_{ext} = \frac{a}{r} v_r Z_o \frac{-ika}{1-ika} \exp\left(ik(r-a)\right)(ik - \frac{1}{r}) \tag{2}$$

The velocity field at an exterior point (x,y,z) and $r = \sqrt{x^2+y^2+z^2}$, is

$$v_x(x,y,z) = \frac{a^2}{r^2} v_r \exp(ik(r-a)) \frac{1-ikr}{1-ika} \frac{x}{r} \tag{3}$$

$$v_y(x,y,z) = \frac{a^2}{r^2} v_r \exp(ik(r-a)) \frac{1-ikr}{1-ika} \frac{y}{r} \tag{4}$$

$$v_z(x,y,z) = \frac{a^2}{r^2} v_r \exp(ik(r-a)) \frac{1-ikr}{1-ika} \frac{z}{r} \tag{5}$$

The analytical expression for the radiated power by a pulsating sphere is given as,

$$W = 2\pi \rho_o c v_r^2 \frac{k^2 a^4}{1+k^2 a^2} \tag{6}$$

5 Results and validation

Acoustic analysis is carried out by running one of the Analysis Sequences defined in the Coustyx MultiDomain model. An Analysis Sequence stores all the parameters required to carry out an analysis, such as frequency of analysis, solution method to be used, etc. In this Demo Model, the analysis is performed at a frequency $f = 54.59 Hz$ using the Fast Multipole Method (FMM) by running "Run Validation - FMM". Coustyx analysis results, along with the analytical solutions, are written to the output file "validation_results_fmm.txt". The results can be plotted using the matlab file "PlotResults.m".

Coustyx uses Direct BE method to solve the radiation problem. In Direct BE method, the primary variables are the pressure and the pressure gradient on the boundary. Field point solutions are then computed from the surface solutions.

The surface pressure on the sphere at point (0, 1, 0) from Coustyx is (210.24, -212.51), which compares well with the analytical solution (207.515, -207.515). Figure 2 shows field point pressure variation with distance from both Coustyx and Analytical methods. The comparisons show very good agreement between the two solutions. The radiated power computed from Coustyx is 1292.53 Watts, which matches well with the exact analytical solution of 1303.86 Watts.

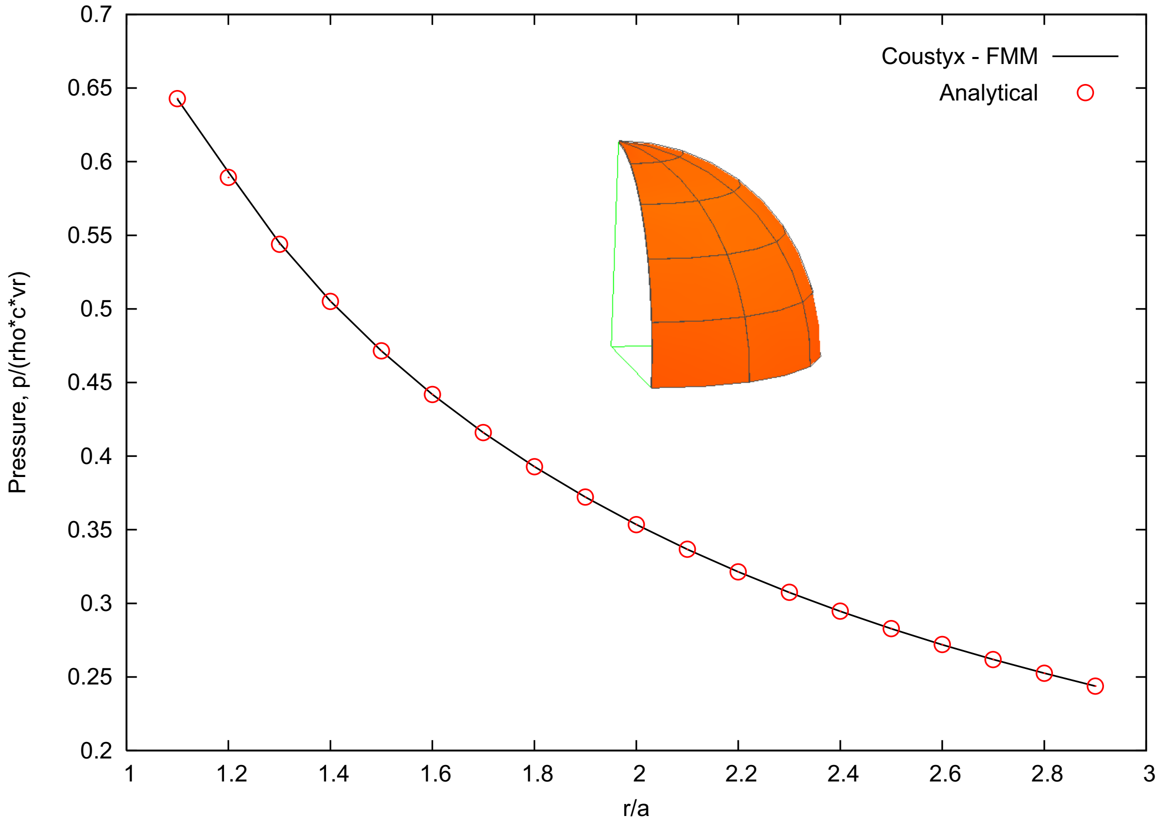

Figure 2: Field point pressure comparisons with distance, r (from center of sphere), from Coustyx and Analytical methods.