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EFFECT OF TOOTH DEFLECTION AND CORNER CONTACT ON BACKSIDE SEPARATION (BACKLASH) OF GEAR PAIRS

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ABSTRACT
Because plastic materials have moduli that are much lower than those of metals, the deflections of loaded plastic gears are much greater than those of steel gears. This paper presents an evaluation of tooth deflections and the effect of load on the backlash of these gears using a finite element program that has an accurate contact deflection analysis embedded within it. In addition to deflection analysis, the effect of tip modification on the contact regime and loads along the edges of the plastic gears is presented. An example spur gear pair and an example helical gear pair are used to demonstrate the analysis methodology. The results of the analysis show that backside tooth contact does not occur as tooth deflections in plastic gears increase with increasing load. In fact, the backside gap actually increases with increasing load.

INTRODUCTION
It has been hypothesized that the large deflections of plastic gears may provide an opportunity for reverse contact due to a reduction in backlash. The main purpose of this paper is to investigate this possibility and at the same time to show the contact mechanisms that result in corner contact [2,4]. The reverse contact mechanism has been mentioned by several practicing engineers but has not been documented in the literature. In this paper we will show that one reason for this lack of documentation may be the fact that it does not seem to occur and in fact, the backlash increases as loads and deflections increase.

Two gear sets, one spur and one helical, are analyzed using the finite element program CALYX developed by Vijayakar [1,3]. This program is specialized for contact analysis of any kind, including gears in two or three dimensions. Analysis can be performed statically or dynamically, but in this paper only three-dimensional static analysis is conducted. Both of the gear sets are run in two situations, one with perfectly involute shapes and the other with linear tip relief on each gear. The possibility of backside contact along with corner contact, load distribution, and pressure distribution are investigated in both cases.

ANALYSIS
Load distribution and backside separation for 10 equally spaced positions in one mesh cycle are evaluated for each gear set. In order to increase the possibility of backside contact, the tooth thickness of the original gear sets is increased so that unloaded backlash is nearly zero. CALYX divides the surface of the gear into contact grid cells in face width and profile direction. Separation between bodies is calculated and the contact solver is activated if separation in any surface drops below a specified value. The reduced backlash allows the backside separation value to be reasonably low so that the backside separation may be quantified. The grid size in the profile direction is an important parameter because it should be wide enough to calculate correct contact pressure. Also, grids that are too wide give inaccurate results because in this case all pressure is handled by only one cell.

Table 1 shows pertinent design parameters of the spur gear set that is analyzed. This spur gear set is run unloaded and under four different load conditions as T_1=0.565, T_2=1.130 T_3=1.695 T_4=2.260 Nm. One mesh cycle is the repeating pattern for angular position of the gears, namely roll angle. A mesh cycle is divided into 10 equally spaced positions, and the first position is named 0.1, last position 1. Figures are plotted for 2.5 mesh cycles for better visualization. The predicted load distribution of the unmodified gear at the rated loading of 1.13 Nm is shown in fig. 1. For this tooth pair, this contact position is particularly unique since it theoretically, has only one tooth pair in contact, yet contact occurs on three tooth pairs. The contact on both the first and last tooth pairs is due to corner contact [2], a tip interference that in this position is occurring simultaneously on both the gear and pinion. This position also gives the highest backside separation. For the backside separation to be seen the graphical output is exaggerated in fig. 2 and fig. 3 so that separation may be visualized when a torque of 0.565 Nm is applied. The exaggeration allows the separation at the backside to be seen, but it is difficult to quantify the backside gap from the figures.

In order to quantify the results, histograms of the contact pressure and backside separation are superimposed on fig. 1 and fig. 3, respectively. The height of the histogram of fig. 3 represents contact pressure amplitude and its width represents the width of the contact zone. In this case we see the amplitude of stress of the corner contact tooth pairs 1 and 3 is extremely high and the contact width is very low when compared with the middle tooth pair. This is due to the low radius of curvature at the corner where contact occurs.

The minimum backside separation may be extracted for the histograms of fig. 3 as the shortest distance from the base of the histogram to its center and is shown as the distance M in the histogram of tooth pair 2. In this case we see position 1, the same position shown in fig. 1.

Table 1: Sample Spur Gear Set Parameters			
	Pinion	Common	Gear
Number of teeth	40		40
Pressure angle		20 ^{o}	
Module [mm]		1.27	
Center distance [mm]		50.8	
Face width [mm]	6.35		6.35
Outside diameter [mm]	53.416		53.416
Root diameter [mm]	46.965		46.965
Tooth thickness [mm]	0.07853		0.07853
Theoretical contact ratio		1.76	
Backlash [μm]		0.5	
Modulus of elasticity [GPa]	2.83		2.83
Rated torque [Nm]		1.130	

Fig. 4 shows the minimum value of backside separation plotted for each position under the five different loading conditions. In the unloaded case, one sees that the backside gap (backlash) is nearly zero. As load is increased, the gap increases and the maximum backlash of 15 micron occurs at position 1, the position shown in fig. 1. The minimum gap occurs at position 0.5, where double tooth contact is about equally spaced about the pitch point of the gear. It is hypothesized that this increase in the backside gap is due to the Hertzian contact deflection between the tooth pairs. In no case is there any indication of backside contact and in fact it is concluded that the backside gap increases with load.

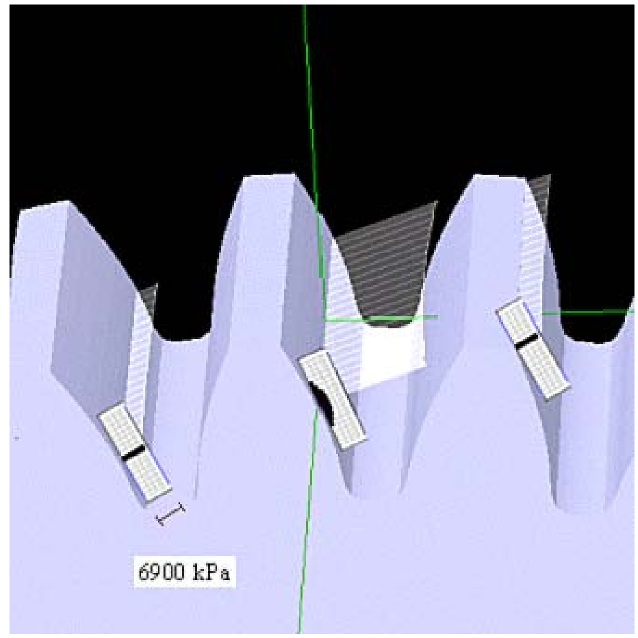

Fig. 1: Load distribution for mesh position 1 of the involute spur gear set at 1.130 Nm torque.

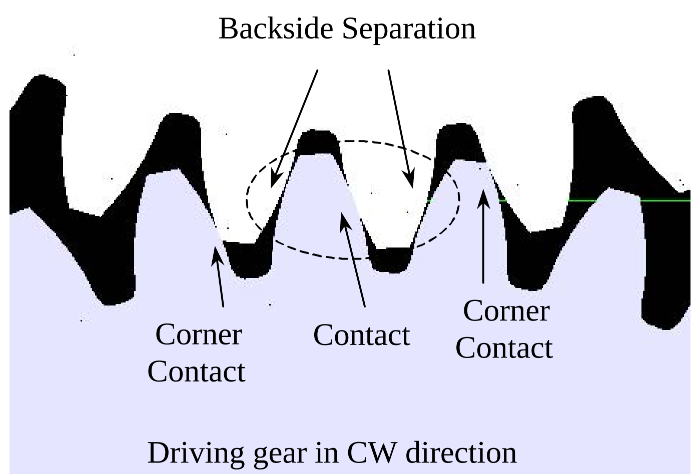

Fig. 2: Graphical view of separation at the backside for 0.565 Nm torque.

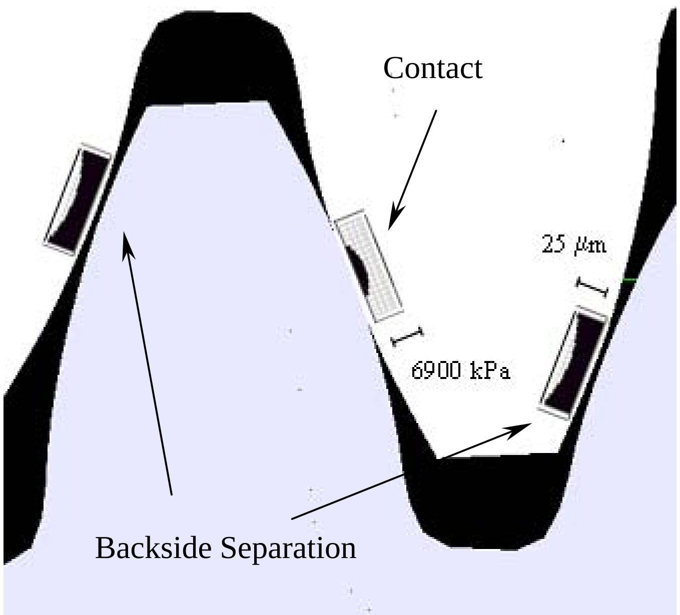

Fig. 3: Expanded view of backside separation in the dashed curve.

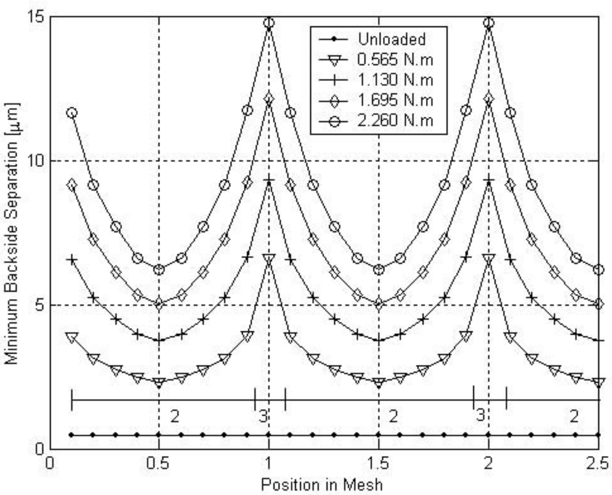

Fig. 4: Map for backside separation.

In fig. 5 the information of fig. 4 is replotted versus load for several positions. In this figure mesh positions through 0.1, and 0.5 are taken along with 1 because separation is symmetric with respect to position 0.5. Here we see that the increase in gap is nearly linear for the lower loads, but becomes less linear for the larger loads, probably due to the advent of the corner contact.

A final byproduct of this analysis is the computation of the transmission error of the gear pair under load. Since this gear pair is a perfect involute, the transmission error will be zero under no load, but takes on values that are proportional to mesh deflection when load is applied. Fig. 6 shows transmission error plotted for two mesh cycles for the load of 1.130 Nm. The number of pairs of teeth in contact is shown along the top of the plot. Here, we see a shape quite similar to the backside separation of fig. 4 except in the region where there are 3 pairs of teeth in contact. The transmission error is more related to the total mesh deflection, whereas backside contact is more an indicator of only the contact deflection, thus providing a rationale for the differences in amplitude and shape of the two plots.

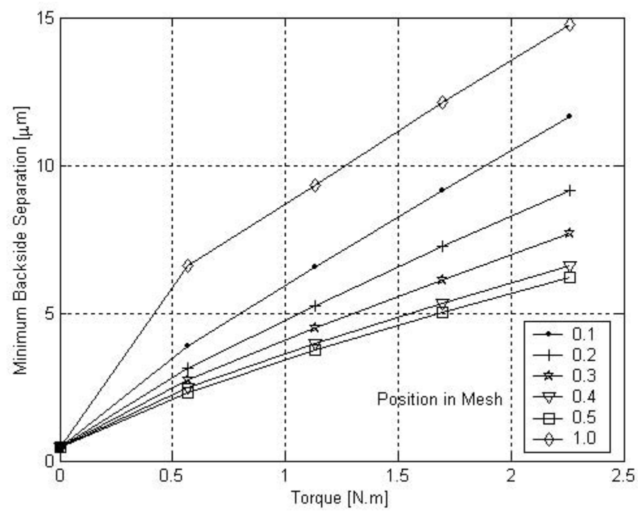

Fig. 5: Separation for each mesh cycle.

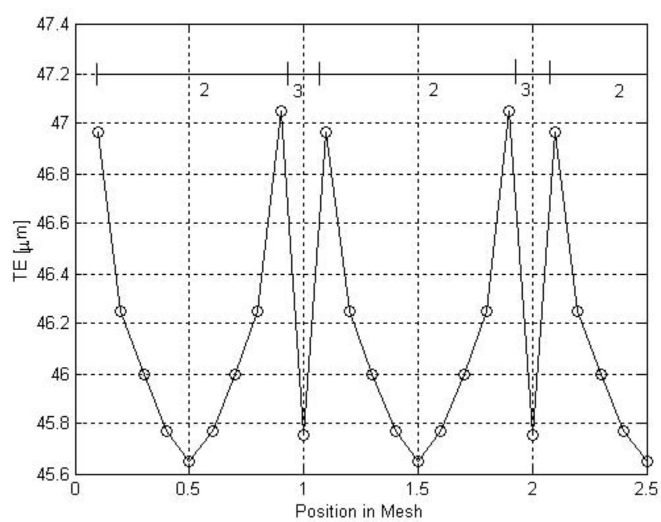

Fig. 6: Transmission Error for unmodified spur gear set at 1.130 Nm torque. Number of teeth in contact shown on top.

Table 2. Modification Data For Sample Spur Gear Set		
	Pinion	Gear
Pitch point roll angle	20.85 °	20.85 °
Tooth tip roll angle	28.77 °	28.77 °
Start of linear modification	24 °	24 °
Amount of modification [μm]	30	30

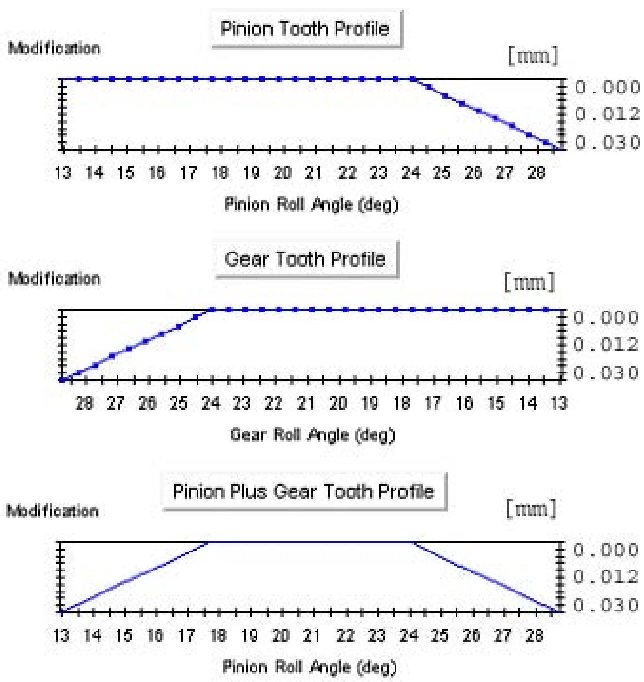

Fig. 7: Linear tip modification data for the spur gears.

Next, a linear tip modification of the form shown in the first two parts of fig. 7 is applied to the gear and the pinion, respectively. Note that the tip of the pinion is at the right side and the tip of the gear is on the left side of the graph and that the sum of the modifications referred to the pinion roll angles is shown on the last graph. The amplitude of the relief is close to the maximum deflection of the tooth pair when in the single tooth pair contact region. Table 2 presents the actual values of the relief that was applied.

Fig. 8 shows the load distribution on pinion for the mesh position 1. Corner contact disappears because of tip relief and the effective contact ratio of this gear pair returns to the theoretical value. This is expected since one of the aims of tip relief is to prevent corner contact. Yet in the above case a contact ratio greater than two is already achieved even though the theoretical contact ratio is 1.76. This is due to corner contact. A tip relief eliminates corner contact, but reduces the actual contact ratio below 2, so that transmission error actually increases. Modification amount is determined to avoid corner contact at the rated load, so that it is possible to have corner contact for increased loading. A question arises regarding the effect of tip modification on the backside gap/contact. Therefore, the earlier analysis for perfect involutes is repeated for the modified set. Separation map for each torque value of the modified spur gear set and the slopes of separation for each position is shown in fig. 9 and fig. 10 respectively. At low loads, there are less number of teeth in contact due to modification, so that it is more convenient to give the contacting teeth number in a separated table, table 3. Also the transmission error at the rated load is plotted in fig. 11.

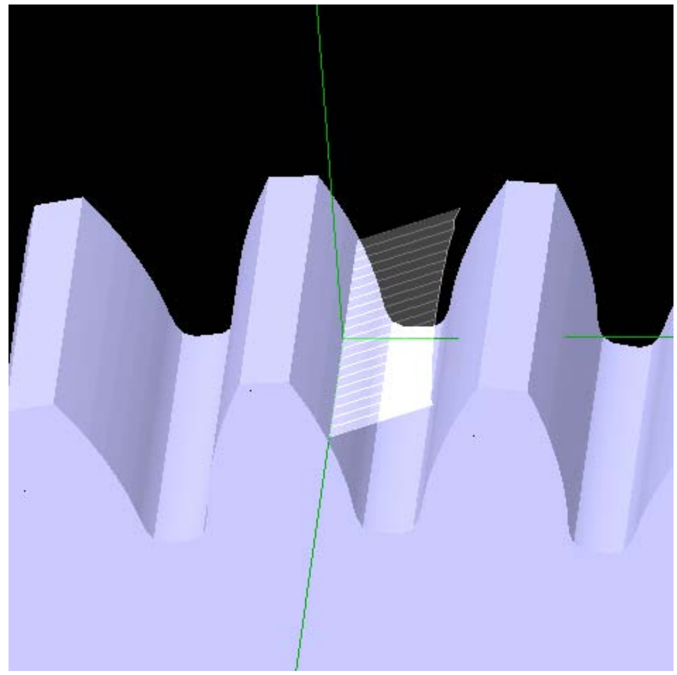

Fig. 8: Load distribution for mesh position 1 of modified spur gear set at 1.130 Nm torque.

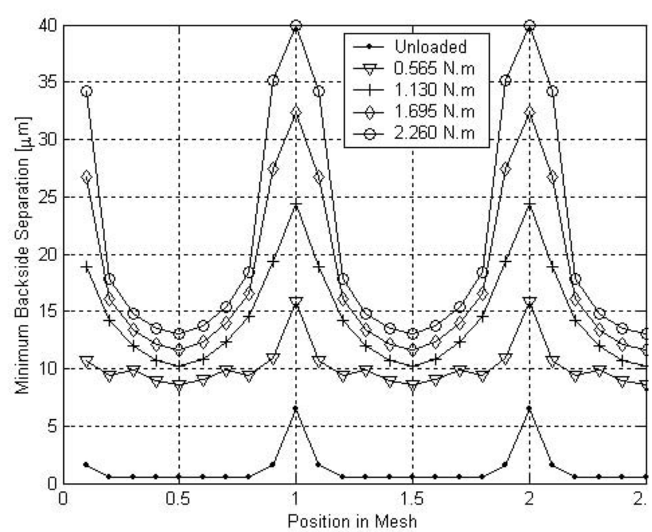

Fig. 9: Map for backside separation for modified set.

Table 3: Number of teeth in contact vs. mesh position and load.										
Torque [Nm]	Position in Mesh									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.565	1	1	1	2	2	2	1	1	1	1
1.130	1	2	2	2	2	2	2	2	1	1
1.695	1	2	2	2	2	2	2	2	1	1
2.260	1	2	2	2	2	2	2	2	1	1

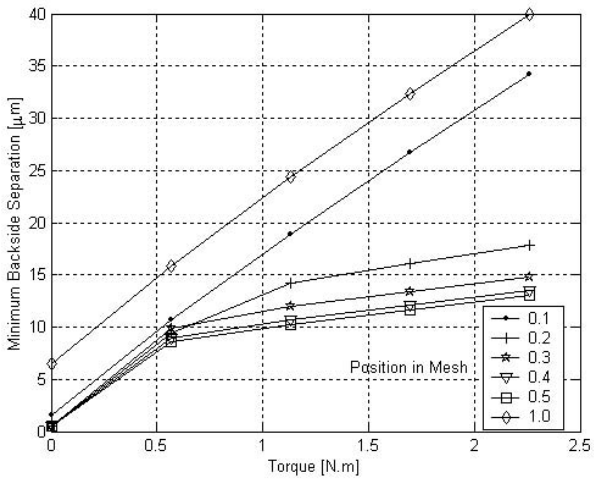

Fig. 10: Separation for each mesh cycle for modified set.

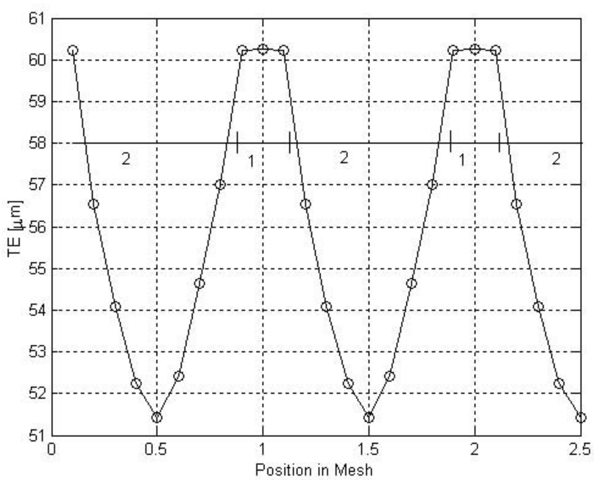

Fig. 11: Transmission Error for modified spur gear set at 1.130 Nm torque.

First, at no load, the tip modification affects the backside separation in the region where the modified portion of the tooth is in contact. At mesh position 1 unloaded backside separation increases as compared to unmodified gear set due to the removal of material from this contact point. Similar to the unmodified set, the rate of change of backside separation with load is different for each position in the mesh cycle. It is possible to see this by comparing the slopes of fig. 5 and fig. 10. At the larger loads, where we have deflections greater than the modification, we again see the effect of the corner contact changing the slope of the plots.

Fig. 11 shows the transmission error shape for 1.130 Nm loading. Here, we have two tooth pairs in contact except at positions 0.1, 0.2 and 1 and the drop in transmission error that occurred earlier when three tooth pairs were in contact now disappears. However, the peak-to-peak value of transmission error is significantly greater, probably due to the fact that the unmodified case has more teeth in contact. Also, some iteration on the shape and amplitude of the tip relief would likely find modifications that would significantly reduce the transmission error value.

Next, a helical gear set with the parameters given in table 4 is analyzed using the same procedures. Fig. 12 shows the load distribution for the helical gear set at mesh position 1 of the mesh at a load of 0.653 Nm. Corner contact now manifests itself as contact extending along the tip of each of the tooth tips as shown in the circles regions. Contact stresses will again be abnormally high in these regions due to the very small radius of curvature at the tooth tips. The backside separation shown in fig. 13 is smoother compared to the spur set but still increases with increasing load. Unlike the spur gear set, the rate of change of separation shown in fig. 14 is same for all positions. Transmission error corresponding to this set is plotted in fig. 15. The peak to peak value is a smaller percentage of the mean value than for the spur set and the sudden changes in transmission error correspond with the numbers of tooth pairs in contact that are shown at the bottom of the graph.

Although the rationale for applying tip relief for helical gears may not be as obvious from the load distribution charts, it is still desirable to use to minimize contact at the tooth tips. Therefore, linear tip modifications, with amplitude enough to avoid corner contact at 0.435 Nm load has been given to both pinion and gear. Actual values of modifications are shown in table 5 and plotted in fig. 16

Table 4. Sample Helical Gear Set Parameters			
	Pinion	Common	Gear
Number of teeth	28		27
Pressure angle		20 º	
Module [mm]		1	
Center distance [mm]		29.875	
Helix angle		23 º	
Face width [mm]	12		12
Outside diameter [mm]	33.047		31.960
Root diameter [mm]	26.866		25.779
Tooth thickness [mm]	1.70635		1.70635
Theoretical contact ratio		3.347	
Backlash [µm]		0.3	
Modulus of elasticity [GPa]	1.64		1.64
Rated torque [Nm]		0.435	

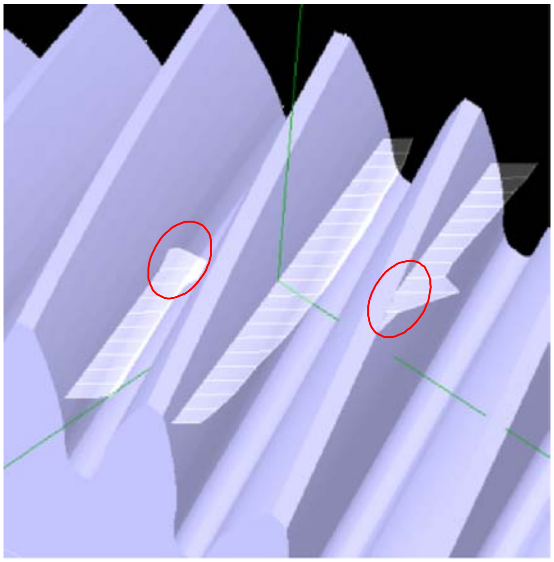

Fig. 12: Load distribution for 10th mesh position of involute helical gear set at 0.653 Nm torque. Corner contact regions are circled.

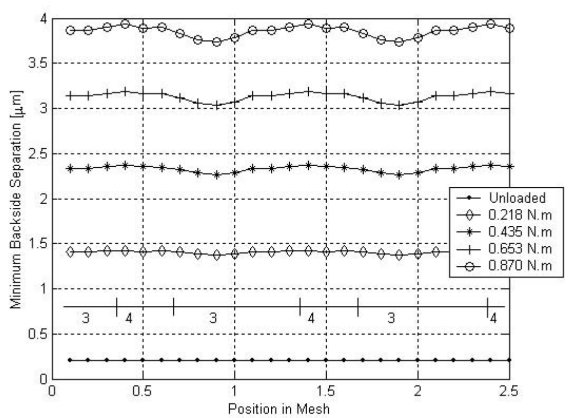

Fig. 13: Map for backside separation for involute helical gear set.

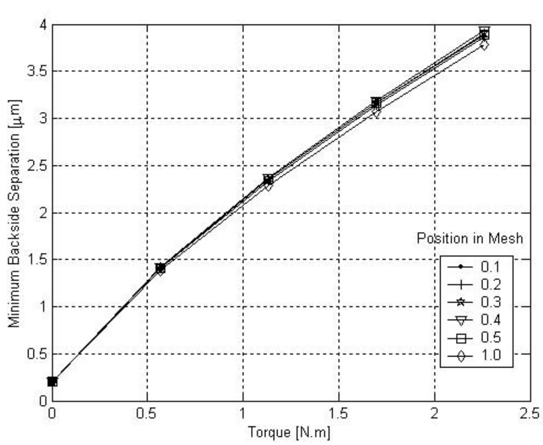

Fig. 14: Separation for each mesh cycle for unmodified helical gear set.

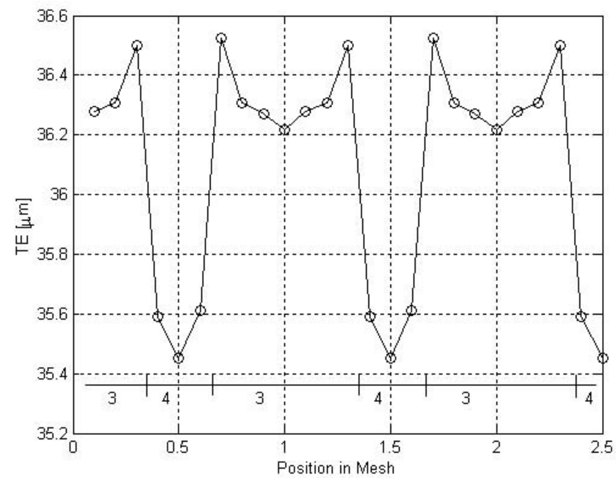

Fig. 15: Transmission Error for unmodified helical gear set at 0.653 Nm torque. Number of teeth in contact shown on bottom.

Table 5. Modification Data For Sample Helical Gear Set		
	Pinion	Gear
Pitch point roll angle	22.66 °	22.66 °
Tooth tip roll angle	34.61 °	34.99 °
Start of linear modification	30 °	30 °
Amount of modification [μm]	13	13

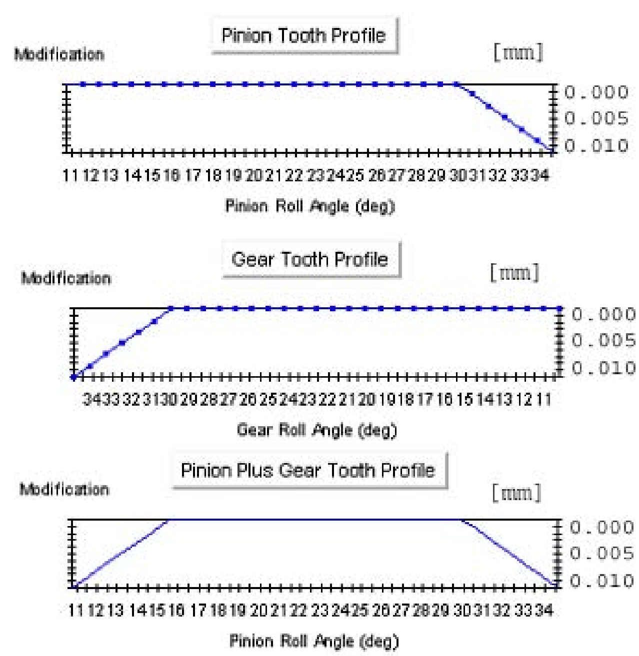

Fig. 16: Linear tip modification data for the helical gears.

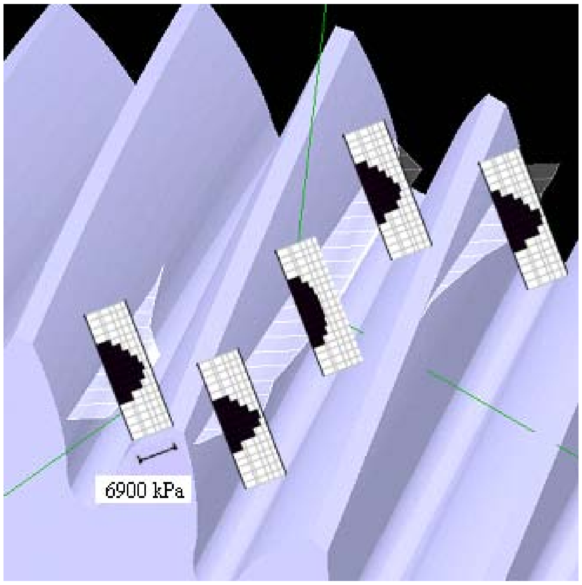

Fig. 17: Load distribution for 10th mesh position of modified helical gear set at 0.435 Nm torque.

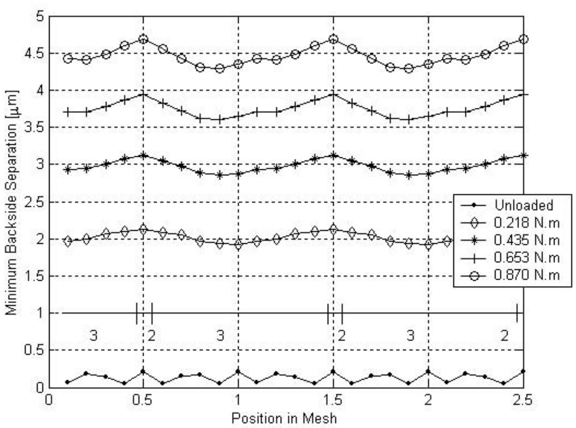

Fig. 18: Map for backside separation for modified helical gear set.

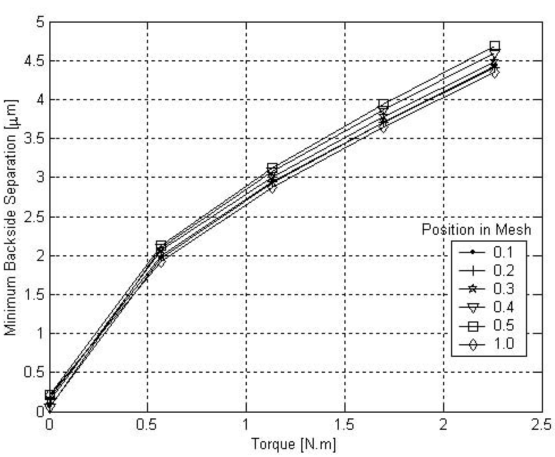

Fig. 19: Separation for each mesh cycle for modified helical gear set.

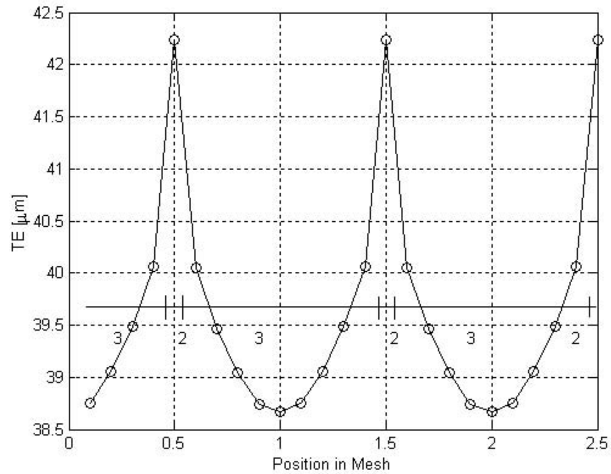

Fig. 20: Transmission Error for modified helical gear set at 0.653 Nm torque.

Figures 17, and 18 show the load distribution, pressure histogram and backside separation map, respectively. The corner contact is almost eliminated, but not completely. Similar to the spur gear set, the amount of separation increased with tip relief. The peak-to-peak transmission error value increased due to the fact that modification reduced the number of teeth in contact from 4 to 2 in corner contact regions. Less number of teeth result in higher deflection than unmodified set. However the rate of change of separation with torque does not vary much unlike the spur gear set. This is also true for the unmodified gear set, and is probably because helical gears have much smoother load distribution compared to spur gears.

Transmission error plot of fig. 20 is in conformity with the number of teeth in contact, where at the corner contact region now contact ratio is 2. Tip relief in terms of transmission error is increasing the peak-to-peak values but in practice it may be desirable. This is because corner contact is not preferable for surface durability reasons.

CONCLUSIONS

We have shown in this paper that backside contact due to tooth deflections will not occur in meshing involute gear pairs. In fact, the backside gap actually increases as load is increased. The addition of tip relief also increases this backside gap.

We have also shown for these examples other derivatives of the analysis, namely, the significance of corner contact and the application of tip relief to minimize this contact.

RECOMMENDATIONS

The authors believe that plastic gear design guidelines and methodologies that utilize tooth deflection as a design performance metric may actually add unnecessary design constraints, and possibly cost, to these types of gear design applications. To address this concern, future work should include the empirical testing of gear sets (such as those presented in this paper) in an effort to correlate analytical backlash results to those actually observed in hardware.

ACKNOWLEDGMENTS

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