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Time-Domain Dynamic Analysis of Helical Gears with Reduced Housing Model

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ABSTRACT

In this paper we present a time-domain dynamic analysis of a helical gear box with different housing models using a unique finite element-contact mechanics solver. The analysis includes detail contact modeling between gear pairs along with the dynamics of gear bodies, shafts, bearings, etc. Inclusion of the housing in the dynamic analysis not only increases the fidelity of the model but also helps estimate important NVH metrics, such as dynamic load and vibration transmission to the base, sound radiation by the gearbox, etc. Two different housing models are considered. In the first, the housing is represented by a full FE mesh, and in the second, the housing is replaced by a reduced model of condensed stiffness and mass matrices. Component Mode Synthesis (CMS) methods are employed to obtain the reduced housing model. Results from both the models are successfully compared to justify the use of reduced housing model for further studies.

Steady state sound radiation by the gear box housing is then studied in the frequency domain using a boundary element solver. The housing frequency response, which is the boundary condition for the acoustic analysis, is estimated using two different methods. In one method, the response is computed from the generalized coordinates and component modes using modal superposition, in the other the bearing dynamic loads are used to perform forced response analysis on the full FE mesh of the housing. Thus, a template for end-to-end solution to predict radiated noise from a gear box is established.

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INTRODUCTION

Gear noise and vibrations remain key concerns in many transmission applications. The dynamic loads or errors at the gear mesh excite the components in a gear box. Typical transfer path for the noise and vibration includes gears, shafts, bearings, and the housing or the base connected to the housing. Low frequency vibrations propagate through structures as structure-borne noise, and higher frequency vibrations typically manifest as air-borne noise. The housing due to its large surface area acts as a good radiator of sound. Thus, it is imperative that the dynamic analysis of the entire gearbox, including the housing, is essential in understanding its noise and vibration characteristics.

Lumped parameter models are commonly used to model gear dynamics, where various components of the gear box are represented by lumped inertias and stiffness. The gear mesh excitation, which is the source for the dynamics, has to be provided externally either as time-varying mesh stiffness or as the static transmission error [1]. General purpose finite element software may also be used to solve gear dynamics. But it requires refined meshes near the contact zone for accurate gear tooth contact modeling. Moreover the local refinement needs to keep moving as the gears rotate, thus making it practically infeasible to solve the time domain gear dynamics problem. In 1991, Vijayakar [2] introduced a combined surface integral and finite element formulation to solve gear contact problems efficiently. The semi-analytical contact formulation along with the superposition of small finite element deflections over large kinematic trajectories of the gears allow one to solve the contact problem in rotating gears fast, thus paving way for solving gear dynamics in reasonable amount of time. Further the need for extremely refined mesh near contact is removed; coarse mesh as shown in Figure (1) would suffice. This formulation has been successfully used to model 2D and 3D gear dynamics by many researchers [3,4,5]. A frequency domain version of this formulation is presented in [7].

The primary focus of this paper is to employ the time-domain FE-contact solver for gear dynamics and use a frequency domain boundary element solver to compute steady state sound radiated by the gear box housing. It might seem more natural to solve a steady state problem in the frequency domain. But, unlike time-domain analysis frequency domain approaches for gear dynamics cannot naturally model the nonlinearities in the system, such as time-varying mesh stiffness, gear tooth contact loss, etc.

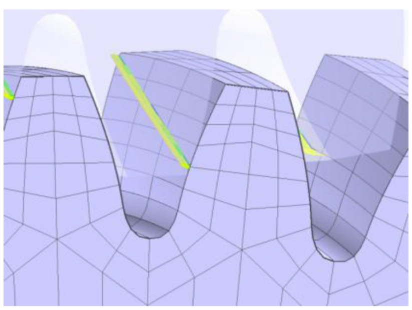

Figure 1. Gear tooth finite element mesh used in the FE-Contact solver. Contact pressure predicted by the solver is also plotted. Note that mesh refinement is not necessary to model contact (in contrast to general purpose FE codes) as the solver uses a combined surface integral and FE formulation [2].

The end-to-end solution to predict noise radiated by the gear box explored in this paper is described in Figure (2). First, we perform the time-domain dynamic analysis of a helical gear box, including the housing, using the same FE-Contact solver discussed above [3,4,5]. Then we estimate the steady state sound radiated by the housing using a boundary element solver presented in [8, 9]. To reduce the solution time the full finite element (FE) mesh of the housing is replaced by a reduced model of condensed stiffness and mass matrices. Component Mode Synthesis (CMS) techniques are used to obtain the reduced housing model. Dynamic analysis is performed for the two housing models. The results are compared to examine the accuracy of the model with the reduced housing.

SIMULATION WORKFLOW

Here we will introduce the simulation workflow followed to predict the noise radiation by a gearbox. Figure (2) shows the summary of the steps involved. Greater details about each step will be covered as we introduce these topics later on in this paper.

Multi-body dynamics model for gear dynamics is built using the unique finite element-contact solver described in [3,4,5]. The inputs for the model include the finite element models of the gears, shafts, bearings and the housing. Full FE mesh of the housing will be replaced by reduced housing model. The time-domain analysis at the operating speed produces many outputs including dynamic loads, deflections, dynamic transmission error (DTE), etc. Among the outputs, the bearing loads or the generalized coordinates of the reduced housing model are used for the acoustic analysis.

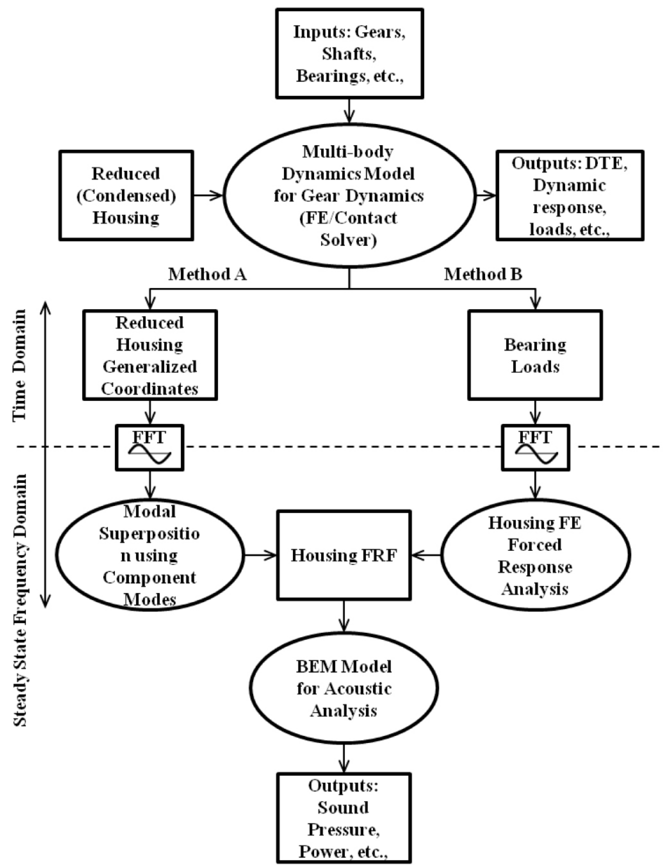

Figure 2. Gear box noise radiation prediction workflow.

Steady state sound radiation by the housing is simulated using a boundary element solver [8] in the frequency domain. The primary input for the acoustic analysis is the housing frequency response function (FRF). This may be obtained by two different methods. In Method A, we utilize the generalized coordinates of the reduced housing model and the component modes to compute housing FRF. In Method B, we use bearing loads to perform forced response analysis on the full FE mesh of the housing. Note that both the generalized coordinates and the bearing loads are first transformed from the time domain to the frequency domain to compute housing FRF. Housing FRF is then used as the input excitation in the boundary element model to solve the housing acoustic radiation problem.

TIME-DOMAIN DYNAMIC ANALYSIS

The time-domain dynamic analysis on the gearbox is performed using the finite element-contact solver especially suited for gear dynamics [3,4,5].

Finite Element Contact Mechanics Model for Gear Dynamics

The only inputs for the finite element contact mechanics model for dynamics are the input torque and speed along with the geometry and material properties of various components of the gear box. There is no need to externally specify the time-varying stiffness or the static transmission error as the excitation to the dynamics model as required for lumped parameter models. These are all intrinsically modeled in the finite element model. The solver uses a combined surface integral and finite element solution approach to model the contact between gears efficiently [2]. Detailed description on the finite element-contact mechanics formulation for 2D gear dynamics is provided in [3]. An extension to the above formulation for 3D gear dynamics is presented in [7].

The equations of motion for gear box assembly are given by

$$\begin{bmatrix} \boldsymbol{M}_{ff} & \boldsymbol{M}_{fr} \\ \boldsymbol{M}_{rf} & \boldsymbol{M}_{rr} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}}_f \\ \ddot{\boldsymbol{x}}_r \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_{ff} & \boldsymbol{C}_{fr} \\ \boldsymbol{C}_{rf} & \boldsymbol{C}_{rr} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}}_f \\ \dot{\boldsymbol{x}}_r \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{ff} & \boldsymbol{K}_{fr} \\ \boldsymbol{K}_{rf} & \boldsymbol{K}_{rr} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_f \\ \boldsymbol{x}_r \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}_f \\ \boldsymbol{f}_r \end{Bmatrix} \quad (1)$$

where $\boldsymbol{x_f}$ represents the finite element deflections with respect to the reference frame attached to rotating gear bodies, $\boldsymbol{x_r}$ is the small rigid body deflections of the reference frame superposed over the large kinematic trajectories of the gears. Rayleigh damping model is used to add damping to gears, shafts and housing, $\boldsymbol{C_{ff}} = \alpha \boldsymbol{M_{ff}} + \beta \boldsymbol{K_{ff}}$. Viscous damping is added through the bearing damping matrices.

The helical gear pair geometry used in this study is adapted from the spur gear pair of involute contact ratio 1.37 from [6], with the addition of the 30 deg. helix angle. Table (1) shows various parameters of the helical gear pair. The shafts, bearings and the housing are arbitrarily picked to build the entire gear box. Figure (3) displays the finite element model with full FE mesh for the housing.

Table 1. Unit gear ratio helical gear pair parameters used in this study.

Number of teeth	50
Module	3 mm
Helix Angle	30 deg
Transverse Pressure Angle	20 deg
Transverse Tooth Thickness	4.64 mm
Center Distance	150 mm
Face Width	20 mm
Outer Diameter	154.41 mm
Inner Diameter	45 mm
Torque	170000 N-mm
Operating Speed	2100 RPM

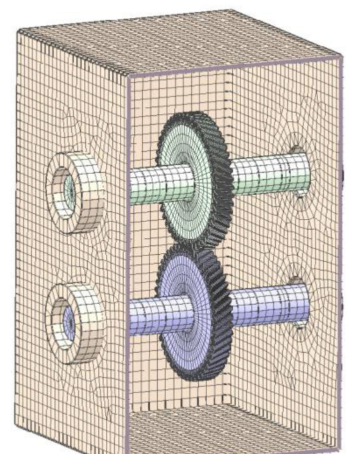

Figure 3. Finite element model of the gearbox with the full FE mesh for the housing (Cut view).

Housing Models

Two different housing models are considered for this study. In the first case a full FE mesh for the housing is used, whereas in the second case the full FE mesh is replaced by a reduced model of housing with condensed matrices. The housing FE mesh chiefly consists of shell elements, except the bearing sleeves with solid elements.

The reduced housing model is obtained by employing the “Craig-Bampton” or “Fixed-Interface” CMS method using a commercial FE software package. CMS methods are used to reduce the size of FE matrices to a smaller size of boundary degrees of freedom and truncated set of normalized mode generalized coordinates [10]. To use Craig-Bampton method we first identify the boundary nodes on the housing FE mesh that are going to interface with the rest of the gear box model. In our case, the four constraint nodes and the nodes that are going to connect to bearings are the boundary nodes. To reduce the number of boundary nodes attached to bearings we created a web of rigid elements connecting boundary nodes to a single node at the center of the bearing sleeve. Therefore we will eventually have four boundary nodes for the four bearing interfaces and four more boundary nodes for the constraints. Figure (4) shows the housing finite element mesh used in the CMS method. We also choose the number of normal modes to truncate the modal basis. Based on these two criterions the finite element software package computes the condensed mass and stiffness matrices using Craig-Bampton method.

The equations of motion for a FE model may be written as,

$$\begin{bmatrix} \boldsymbol{M}_{BB} & \boldsymbol{M}_{BI} \\ \boldsymbol{M}_{IB} & \boldsymbol{M}_{II} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}}_B \\ \ddot{\boldsymbol{x}}_I \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{BB} & \boldsymbol{K}_{BI} \\ \boldsymbol{K}_{IB} & \boldsymbol{K}_{II} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_B \\ \boldsymbol{x}_I \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}_B \\ 0 \end{Bmatrix}$$

or,

$$\boldsymbol{M}_H \ddot{\boldsymbol{x}}_H + \boldsymbol{K}_H \boldsymbol{x}_H = \boldsymbol{f}_H \tag{2}$$

where the degrees of freedom associated with the boundary nodes are represented by ***B*** and the interior nodes by ***I***.

The physical coordinates of the housing $\boldsymbol{x_H}$ may be represented by the component generalized coordinates $\boldsymbol{x_h}$ which is a vector of boundary degrees of freedom $\boldsymbol{x_B}$ and a set of truncated generalized modal coordinates $\boldsymbol{\eta}$ as follows,

$$\boldsymbol{x}_H = \begin{Bmatrix} \boldsymbol{x}_B \\ \boldsymbol{x}_I \end{Bmatrix} = \boldsymbol{T} \begin{Bmatrix} \boldsymbol{x}_B \\ \eta \end{Bmatrix} = \boldsymbol{T}\boldsymbol{x}_h \tag{3}$$

where ***T*** is the transformation matrix.

Equation (3) is important for back-computing the physical coordinates once the component generalized coordinates are known.

The condensed stiffness and mass matrices are given by

$$\boldsymbol{M}_h = \boldsymbol{T}^T \boldsymbol{M}_H \boldsymbol{T} \qquad \boldsymbol{K}_h = \boldsymbol{T}^T \boldsymbol{K}_H \boldsymbol{T} \tag{4}$$

In Craig-Bampton method, the transformation matrix ***T*** is computed from the redundant static constraint modes, $\boldsymbol{S_{IB}}$, and the fixed-interface normal modes, $\boldsymbol{\Phi_I}$ (these are the eigen vectors computed with interface nodes fixed) [10].

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{I} & 0 \\ \boldsymbol{S}_{IB} & \Phi_I \end{bmatrix} \tag{5}$$

where,

$$\boldsymbol{S}_{IB} = -\boldsymbol{K}_{II}^{-1} \boldsymbol{K}_{IB} \ ,$$

and ***I*** is an identity matrix.

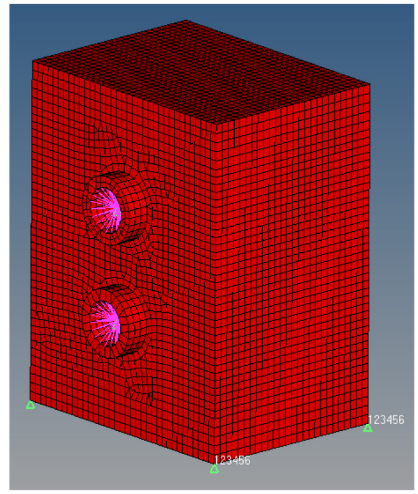

Figure 4. Mesh used in the component mode synthesis technique to create the reduced housing model. At each bearing sleeve a web of rigid elements (shown in pink) are added to connect bearing interface to a single boundary node. Constraint nodes that are considered as the boundary nodes are also shown.

Dynamic Analysis Results & Discussion

Time-domain dynamic analysis is performed at gear speed 2100 RPM or 35 Hz. The gear mesh frequency at this speed is 1750 Hz. The bearing reactions for the two models with different housing models are compared in Figures (5) and (6). The transients are excited due to the step input of torque. At time zero the torque applied on the system is zero and at the next time step operating torque is applied. Sufficient number of time steps is chosen for the system to reach steady state. The time step is chosen small enough to capture frequencies up to the 5th mesh frequency harmonic. However we deal with only the frequencies within the first two mesh frequency harmonics in this study.

Figure (5) shows the transient bearing reactions from the two cases with different housing models. Both the time and frequency domain comparisons match well. The discrepancy in the frequency spectrum for frequencies above 4000 Hz could be attributed to the truncation of normal modes while building the reduced housing models.

The steady state bearing reactions from both the housing models are shown in Figure (6). The time domain and frequency domain comparisons match very well. As expected we find gear mesh frequency and its higher harmonics in the frequency spectrum of the steady state response. The first two mesh frequency harmonics at f_1=1750 Hz, and f_2=3500 Hz are displayed.

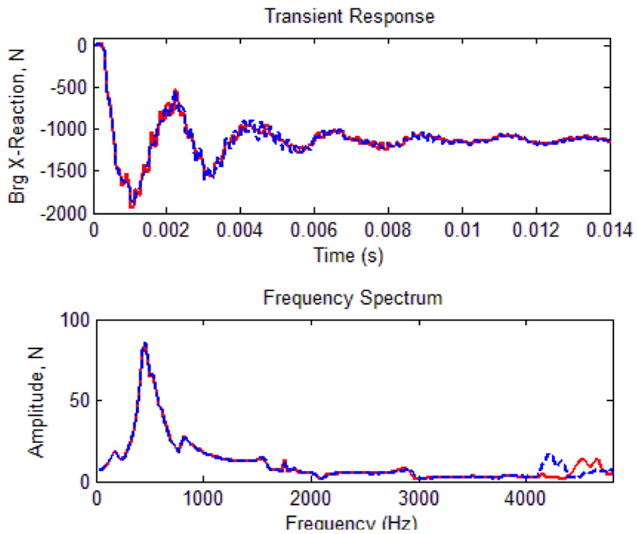

Figure 5. Transient gear shaft bearing 2 X-reaction from the time-domain dynamic analysis of the gearbox. Red solid line correspond to the model with the housing represented by the full FE mesh, Blue dashed line correspond to the model with the reduced housing.

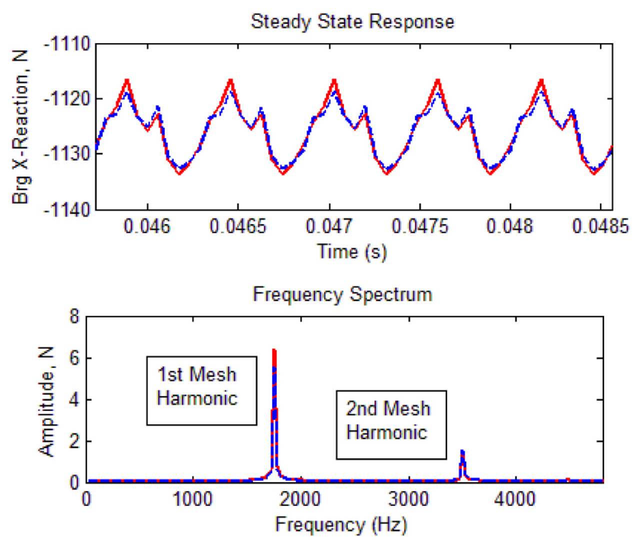

Figure 6. Steady-state gear shaft bearing 2 X-reaction from the time-domain dynamic analysis of the gearbox. Red solid line correspond to the model with the housing represented by the full FE mesh, Blue dashed line correspond to the model with the reduced housing.

Thus we establish that the reduced housing model may be effectively used in place of the full FE mesh of the housing. From this point forward we will use the dynamics results from the analysis with the reduced housing model.

STEADY STATE ACOUSTIC ANALYSIS

A frequency domain boundary element solver [8, 9] is used to estimate the air-borne noise radiated by the housing under steady state conditions. The steady state results from the above time-domain dynamic analysis are transformed to the frequency domain to be used for the acoustic analysis.

Housing Frequency Response Function (FRF)

We require the housing frequency response function (FRF) to solve the acoustic radiation problem. Below two different methods are detailed for computing the housing FRF.

Method A: Housing FRF from component modes superposition

In this method we utilize the generalized coordinates of the reduced housing model and the component modes to compute housing FRF. First, the steady state generalized coordinates of the reduced housing model are transformed to the frequency domain. Then, we perform modal superposition using the component modes and generalized coordinates. From equation (3) we obtain the response on the entire housing FE mesh. Please note that we need to have the component mode matrix to use this method.

Method B: Housing FRF from forced response analysis using bearing loads

In this method we use bearing loads to perform forced response analysis on the full FE mesh of the housing. The steady state bearing loads from the time-domain analysis above are transformed to the frequency domain and are applied as forced in housing forced response analysis.

Boundary Element Acoustic Model

Boundary element methods are well suited to solve exterior acoustic radiation problems. The main advantage comes from the requirement of discretizing only the surface of the vibrating structure and not the acoustic domain itself. We employ a powerful BEM solver accelerated by fast multipole method (FMM) [8].

Acoustic Analysis Results & Discussion

The acoustic analysis is performed only at the frequencies related to the first (f_1=1750 Hz) and second (f_2=3500Hz) mesh frequency harmonics as the steady state response is dominated by these frequency components. Figure (7) shows the sound pressure level contours over the surface of the housing and over a spherical field mesh at the frequency 1750 Hz. Figure (8) compares the radiated sound power computed from methods A and B. The differences between the two methods may be attributed to the truncation of modes in method A as compared to method B.

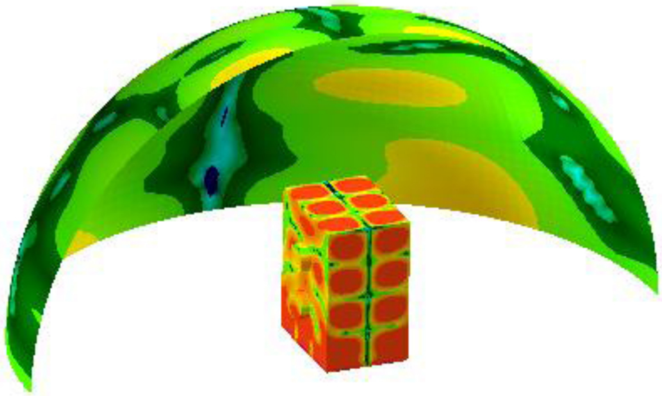

Figure 7. Sound pressure level contour plot over the surface of the housing and a spherical field mesh at the first mesh frequency harmonic (f_1=1750 Hz).

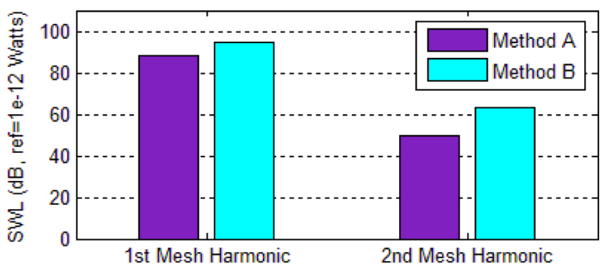

Figure 8. Radiated sound power levels (SWL) by the housing at the first two mesh frequency harmonics (f_1=1750Hz, f_2=3500 Hz).

SUMMARY/CONCLUSIONS

An end-to-end solution to predict radiated noise from a gear box is presented. Unique finite element contact mechanics solver is used to model the gear dynamics. Reduced housing model is used to speed up the analysis. Results from the analysis with reduced housing model match well with the analysis with full FE housing to commend their use for future analysis. Housing FRF is computed from the steady state response and is used to solve the acoustic radiation problem using a boundary element solver. Two different methods used to compute housing FRF give different levels of accuracy for the sound radiated by the housing.

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DEFINITIONS/ABBREVIATIONS

NVH - Noise Vibration and Harshness

CMS - Component Mode Synthesis

FE - Finite Element

FRF - Frequency Response Function

BEM - Boundary Element Method

FMM - Fast Multipole Method