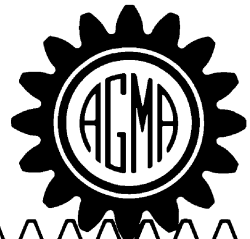


# A Method to Define Profile Modification of Spur Gear and Minimize the Transmission Error

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**American Gear Manufacturers Association**



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**TECHNICAL PAPER**

# **A Method to Define Profile Modification of Spur Gear and Minimize the Transmission Error**

**M. Beghini, F. Presicce, C. Santus, Dipartimento di Ingegneria Meccanica, Facoltà di Ingegneria Università di Pisa, Italy**

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

## **Abstract**

The object of this article is to propose a simple method to reduce the transmission error (T.E.) for a given spur gear set, at the nominal torque, only by means of the profile modification parameters. Iterative simulations with advanced software, are needed. A hybrid method has been used, combining the Finite Element technique with semianalytical solutions. A 2D analysis is thought to be adequate for this kind of work, in fact, this software requires both little time for model definition and simulations, with a very high precision of the results.

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# A Method to Define Profile Modification of Spur Gear and Minimize the Transmission Error

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## Introduction

In this paper a simple method to minimize the Peak to Peak Transmission Error (*PPTE*) for a spur gear set is proposed.

A parametric analysis using advanced software was performed to obtain a general understanding of the problem. Optimization variables were reduced to the Start Relief Roll Angles.

A parabolic topography was assumed for the tip relief and either Low Contact Ratio (LCR) or High Contact Ratio (HCR) spur gears have been considered. Meshing related constraints on optimization have been taken into account. On the basis of parametric analysis an effective optimization procedure has been suggested for LCR that can be extended even for HCR.

## Nomenclature

$P$	Diametral Pitch
$b$	Face width
$h$	Tooth height
$s$	Coordinate along profile
$TE$	Transmission Error
$PPTE$	Peak to Peak TE
$\theta_P$	Pinion start relief roll angle
$\theta_G$	Gear start relief roll angle
$v_e^{Pinion}$	Pinion relief amount
$v_e^{Gear}$	Gear relief amount
$v_e$	Total relief amount
$SAP$	Start Active Profile roll angle
$EAP$	End Active Profile roll angle

Table 1. Nomenclature.

## 1 Transmission Error Effect on Gear Noise

In a gear set, Transmission Error (*TE*) is defined as the difference between the effective and the ideal position of the output shaft with reference to the input shaft. The ideal position represents a condition of perfect gear box, without geometrical errors and deflections. *TE* can be expressed either by an angular displacement or, more conveniently, as a linear displacement measured along a line of action at base circle according to [1]. *TE* is typically considered to be the primary cause of whining noise.

Indeed, whining noise is produced by:

- changing tooth load amplitude,
- changing load position along the tooth profile,
- changing tooth load direction.

These changes are consequences of tooth deflection, local contact deformation and body deformation quantities, which are the origin of *TE*. Several Authors [2-6] studied the correlation between *TE* and profile modification, in order to reduce the *TE*. Niemann [4] proposed *long* and *short* modifications.

This different denomination is based on the start point of tip relief modification along the profile.

According to experimental results, gears with long modification have the minimum *PPTE* and the minimum noise level at the design torque. At lower torque this optimum condition was not verified and an intermediate or short modification is suggested. In these works *short* and *long* modifications are considered to be technological limits and it is not useful to extend the relief to the pitch radius, as described in [1,3].

In the present paper, instead, the start tip relief modifications ( $\theta_P$ ,  $\theta_G$ ) have been considered as design variables ranging from Start Active Profile roll angle (*SAP*) up to the End Active Profile roll angle (*EAP*). Total relief  $v_e$  (at the tooth tip) is imposed equal to the deflection of tooth pairs under the nominal torque. The topography of the tip relief is parabolic. The conditions for obtaining the minimum *PPTE* configuration are found and discussed.

## 2 Methodology

The quantities  $v_e^{Pinion}$  ( $v_e^{Gear}$ ) and  $\theta_P$  ( $\theta_G$ ) are defined in figure 1.

The ranges for the two Roll Angle variables are the Start Active Profile roll angle (*SAP*) and the End Active Profile roll angle (*EAP*) for each gear (figure 2).

Meshing gears simulations have been carried out by means of a hybrid method, combining the Finite Element technique with a semi-analytical solution [7,8].

The main assumptions for the analysis are the following:

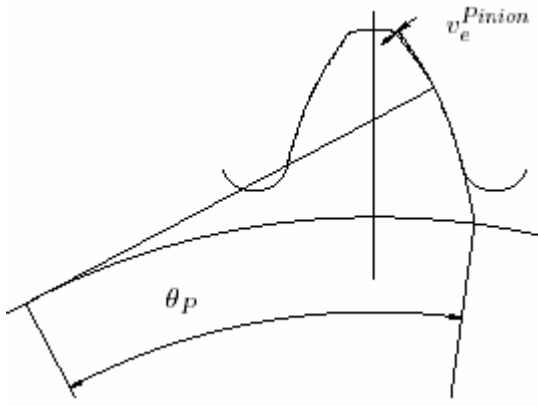


Figure 1. Definition of  $v_e^{Pinion}$  ( $v_e^{Gear}$ ) and  $\theta_P$  ( $\theta_G$ ).

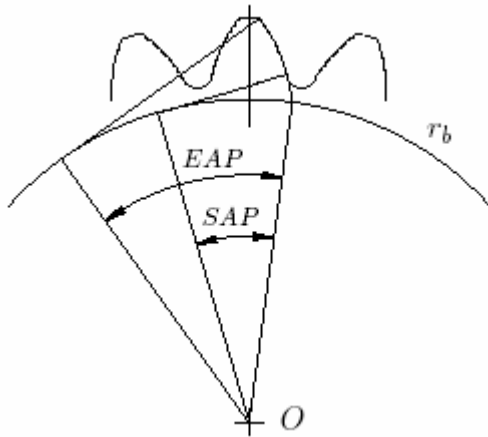


Figure 2. Symbols  $SAP$  and  $EAP$ .

**Plain strain conditions:** suggested by the spur gear geometry (high ratio  $b/h$  and so not affecting bending deformation behavior). 2D plane strain analysis is adequate for this kind of tooth. Moreover the bi-dimensional version of the software requires little time both for model definition and simulations, with a very high precision of the results.

**Static analysis:** static  $TE$  was determined neglecting rotational speed and inertia forces.

**Friction neglected:** in order to get rid of the uncertainty about Coulomb frictional coefficient. The friction force is assumed not affecting the  $TE$  shape function substantially.

**Spacing error and pitch error not considered:** the nominal  $TE$  was calculated, and no statistical consideration have been performed in the analysis. Otherwise required if geometrical errors considered.

Two kinds of spur gear set have been analyzed, in which the  $TE$  function is rather different:

- Low Contact Ratio gears ( $LCR$ ),
- High Contact Ratio gears ( $HCR$ ).

For the  $LCR$  gear contact ratio is between 1 and 2 and when the contact is at the pitch position, only

one tooth pair is in contact, while for  $HCR$ , 3 tooth pairs are in contact at the pitch position.

At the beginning each spur gear set was analyzed by assuming no modification. The maximum deflection of the gear pair tooth,  $TE_{max}$  (evaluated along the pinion base circle) was then calculated and imposed equal to the total relief amount.

$$v_e = TE_{max}$$

The amount of the tip relief was equally shared between pinion and gear assuming *long* modification suggested in [4,5].

$$v_e^{Pinion} = v_e^{Gear} = \frac{v_e}{2}$$

Successively, contact pressure was calculated along the meshing zone in order to detect possible corner contacts.

As shown in figure 3 the corner contact can be observed at both sides.  $v_e^{Gear}$  has to be increased in order to avoid corner contact at the start of meshing, likewise  $v_e^{Pinion}$  has to be increased to avoid the corner contact at the end of meshing.

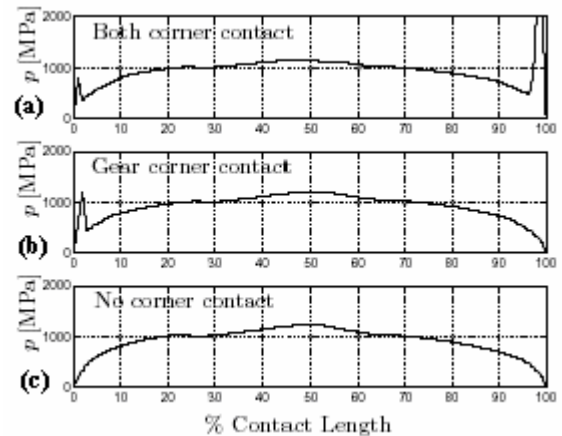


Figure 3. Adapting  $v_e$  to avoid Corner Contacts: (a) Corner Contact both at Start and End meshing, (b) Corner Contact only at Start meshing, (c) No Corner Contact detected.

When corner contact was eliminated, pinion relief ( $v_e^{Pinion}$ ) and gear relief ( $v_e^{Gear}$ ) were held fixed through the subsequent calculations.

To perform the minimization, an object function has to be defined along the variables and the ranges in which the optimum is searched.

In a similar analysis [9] the object function was related to the Fourier expansion of the  $TE$  and the first three harmonics considered. However as shown in figures 4 and 5, in this analysis the first three harmonics could not be the main part of the signal reproducing the  $TE$  function.

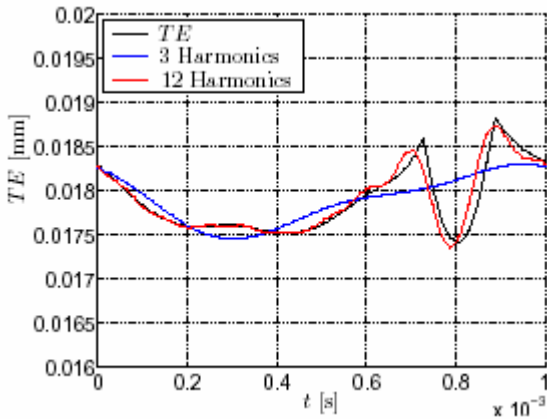


Figure 4. Decomposition of the PPTe according to FFT and signal reproduction.

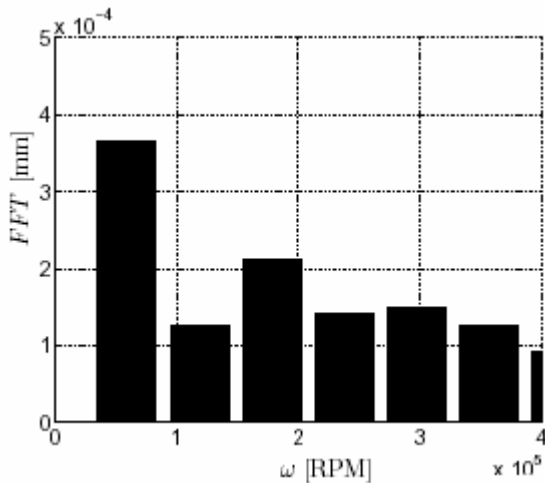


Figure 5. FFT harmonics components of total  $TE(t)$  signal

The reason is that at the instances when the contact passes from different tooth pairs the  $TE$  is not regular ( $C^0$  condition satisfied only). Thus according to these considerations the  $PPTe$  was considered as the object function, and the independent variables were reduced to the start Roll Angles for gear and pinion  $\theta_P, \theta_G$ . The input torque applied to the pinion is the nominal value of the mission profile. For every configuration of  $(\theta_P, \theta_G)$  the following outputs were considered:

- Transmission Error,
- tooth load,
- contact pressure,
- bending principal stress at tooth root.

Each of them were calculated as a function of the Contact Length along the meshing zone. The following limits were then imposed as boundaries of the optimization domain:

**Corner Contact.** For particular starting relief Roll Angle combinations  $(\theta_P, \theta_G)$  corner contact can reappear even if total relief amount is set as discussed above.

**Contact Pressure.** Due to profile modification, relative curvatures are modified and contact pressure can be locally increased, exceeding the *Pitting* limit,

**Bending Stress.** Principal stress at the tooth root can exceed the *Fatigue* strength limit.

Each of the boundary are here better discussed.

**Corner Contact.** Corner contact is produced when the contact region includes zones of the fillet of the tooth tip and the contact pressure rises locally at the tip fillet, as in figure 6.

Furthermore as a consequence of the teeth deflection the effective contact ratio is greater than that found according to rigid geometry hypothesis and so corner contact can be detected in instances when the contact should not appear (figure 7).

This definition of *Corner Contact* can be exploited only if a Loaded Tooth Contact Analysis (LTCA) is performed.

When the corner contact is detected, the calculated pressure peak was not considered reliable, as in this situation the maximum is strongly affected by the radius of the fillet [10], which is a very unpredictable quantity for its technological generation.

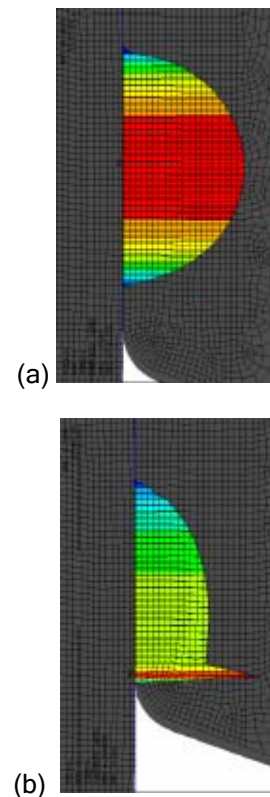


Figure 6. (a) No Corner Contact detected. (b) Corner Contact detected

It is worth noting that starting from long profile modification by lowering Pinion and Gear relief Roll Angles  $(\theta_P, \theta_G)$  even if amounts  $(v_e^{Pinion}, v_e^{Gear})$

are kept fixed, corner contact can reappear. This is due to the fact that if pinion and gear tooth flanks in contact are both modified, the effective total relief is lessened. To understand this an example is given by the green curve of the third graph of figure 8.

In that situation the nominal relationship between Pinion and Gear Roll Angles has been claimed, but according to the aforementioned figure 7 the teeth deflection generates angular shift which is related to the torque applied, and then earlier tip relief overlap has to be expected. This is the cause of Corner Contact reappearing.

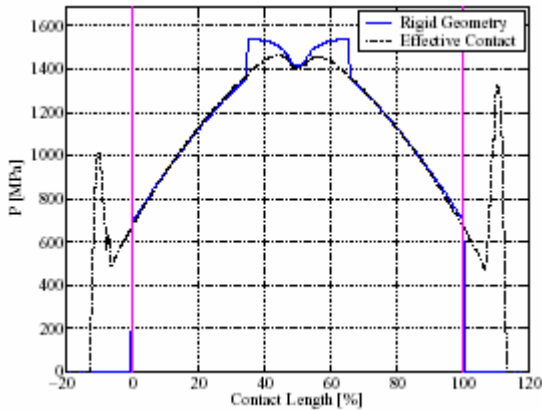


Figure 7. Comparison between effective Contact Pressure and rigid geometry Contact Pressure estimation. The strong contact length enlargement is here depicted.

Configurations producing Corner Contact were obviously considered outside the boundary of the minimization domain.

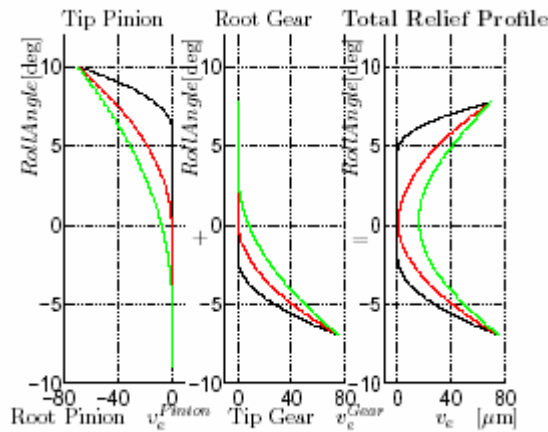


Figure 8. Examples of three different Tip Relief profiles' combinations.

**Contact Pressure.** Local contact pressure along the profile, can change considerably due to profile modifications. Actually, by changing the Start Relief point along the profile, relative curvature considerably changes as well.

In figure 9 it is shown a contact pressure rise with the curvature discontinuity inside the contact region. This numerical result has been confirmed theoretically in [10].

A worse condition (continuity  $C^0$  condition satisfied only) is found when the topography is linear [11] but it is not treated in this paper.

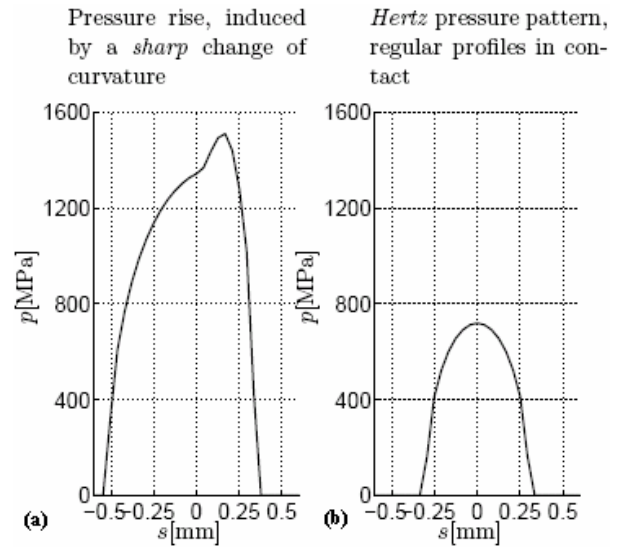


Figure 9. (a) Contact pressure with rise of curvature, on the right. (b) Contact pressure in the subsequent tooth pair, with no curvature discontinuity inside the contact region.

**Bending Stress.** Bending stress at tooth root is another important issue in high performance gear design. By applying profile modifications, load transfer between teeth pairs, contact points and load directions can change. As a consequence, different bending stress  $\sigma_1$  at tooth base can be produced.

If a LCR gear set were considered, bending stress variations would not be greater than 5%, as shown in figure 10, however for HCR gear set variation is greater because with three tooth pairs in contact the sharing factor is more sensitive to profile modification.

Anyway it can be convenient to consider bending stress as a possible penalty for the object function instead of a boundary.

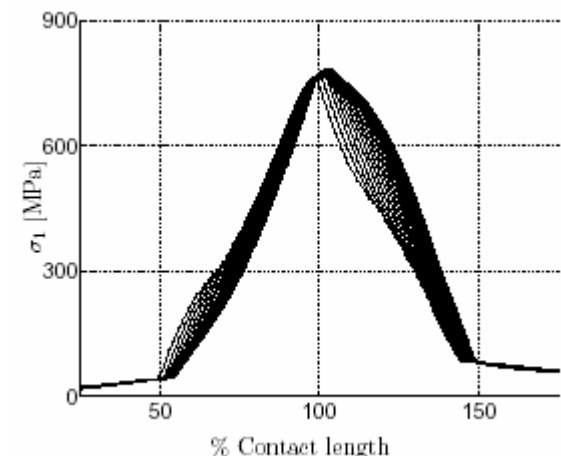


Figure 10. Effect of different profile modification on bending stress on a LCR gear set.

## 2.1 Computational performances

To perform the minimization analysis, following the steps presented in the previous section, a hardware platform *PC* was used with the following characteristics

- CPU 2.6 GHz
- RAM 1 GB

Plane strain analysis was performed by an advanced hybrid *FEM* analysis software, whose references are [12,13].

Analysis were automatically performed in about 12 CPU hours, simulating 50 time steps for each meshing, for 600 different relief  $(\theta_P, \theta_G)$  configurations

### 3 Results

#### 3.1 LCR spur gear set

The analyzed LCR gear set design parameters are summarized in table 2.

Pinion N. of teeth	80
Gear N. of teeth	80
P	0.571 [mm <sup>-1</sup> ]
Pressure angle	22.5 [deg]

Table 2. LCR gear set parameters.

The object function *PPTE* for the LCR gear set is shown in figure 11. No boundaries are presented yet.

It is worth noting that:

- The *PPTE* minimum is unique inside the ranges for the two variables.
- Near the minimum the Hessian matrix is positive defined.

According to these conditions the minimum could be found by adopting a classical gradient derived method.

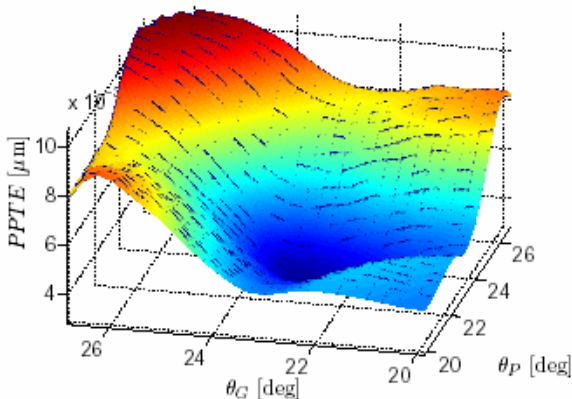


Figure 11. Three-dimensional plot of the *PPTE*, LCR gear set.

As pinion and gear have the same number of teeth, meshing properties are symmetric about the domain diagonal defined by the equation  $\theta_P = \theta_G$  and the absolute minimum is on this diagonal (figure 12).

The numerical values are the following:

$$\begin{aligned} \theta_P^{\min} &= 23.035 \text{ deg} \\ \theta_G^{\min} &= 23.035 \text{ deg} \\ PPTE^{\min} &= 1.8 \text{ }\mu\text{m} \end{aligned}$$

The *TE* with minimum *PPTE* is shown in figure 14. In order to analyze this minimum, *TE* functions for  $\theta_P = \theta_P^{\min}$  at different  $\theta_G$  are plotted. It is remarkable that the shape of the *TE* functions is different between configurations with  $\theta_G < \theta_G^{\min}$  and  $\theta_G > \theta_G^{\min}$ . This is due to the fact that for  $\theta_G < \theta_G^{\min}$ , profile configurations cause a  $v_e$  drop. In fact in the analyzed configuration, start relief roll angles couple  $(\theta_P, \theta_G)$  for the minimum *PPTE* produces overlapping modified profiles and a reduced effective total amount relief as already discussed and shown in figure 13.

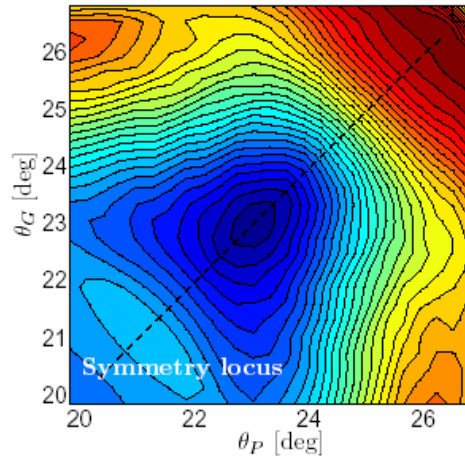


Figure 12. Symmetry locus of *PPTE* $(\theta_P, \theta_G)$  function for LCR gear set.

This is due to an overestimation of the total relief amount calculated at the beginning. For example in this configuration the total amount is reduced from 25  $\mu\text{m}$  to 23.3  $\mu\text{m}$ . And this result can be an insight for better evaluating  $v_e$ .

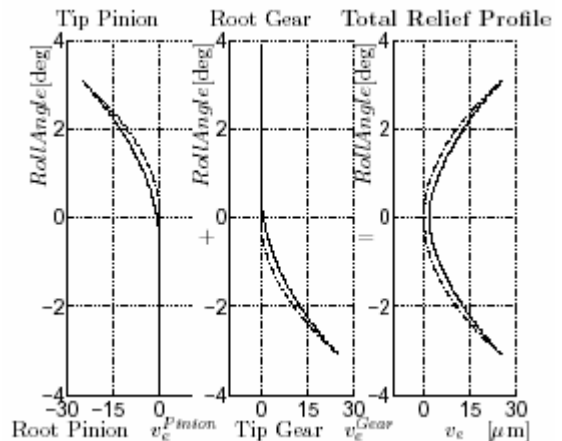


Figure 13. Effective relief diagram, with evidence to the relief overlap and its lessening effect on total relief.

#### 3.1.1 Application of boundaries to optimum profile search

By taking into account the boundaries previously discussed, it was found that the absolute *PPTE* minimum is outside the acceptance limits. As shown in figure 15, the minimum *PPTE* would



produce *Corner Contact*. This is due to the start relief roll angle overlapping observed at the end of the previous section that reduced effective total relief amount.

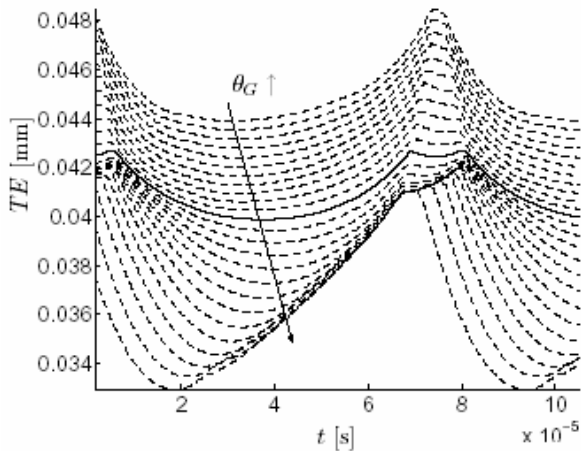


Figure 14.  $TE$  at different  $\theta_G$  for  $\theta_P = \theta_P^{min}$  and minimum  $PPTE$  emphasized.

So instead of the absolute minimum, an approximation inside the boundaries has to be indicated. It is worth noting that along the line referred to the condition of *Consecutive Roll Angle*

*Locus* (black dashed line of figure 15) the  $PPTE$  can be found very close to the absolute minimum.

The Consecutive Roll Angle Locus is defined as start relief Roll Angles ( $\theta_P, \theta_G$ ) that for all meshing contact points either the Pinion or the Gear tooth flank is modified (as in dashed line of figure 13).

Due to definition of Roll Angles ( $\theta_P, \theta_G$ ) and involute properties, the Consecutive Roll Angle Locus is a straight line in figure 15.

With Consecutive Start Relief Roll Angles after imposing the total amount  $v_e$ , only one optimization variable can be considered instead of two, and then a great simplification of the minimization algorithm is produced. A good approximation of the  $v_e$  can be found by exploiting the method presented above (see figure 3) as little modification of  $PPTE$  function was found for different  $v_e$  around this value.

This method was successfully tested for other *LCR* spur gear sets with different numbers of teeth. It is remarkable that for equal teeth number the minimum along the Consecutive Roll Angle Locus is simply in the middle of the graph for symmetry sake (as shown in figure 15).

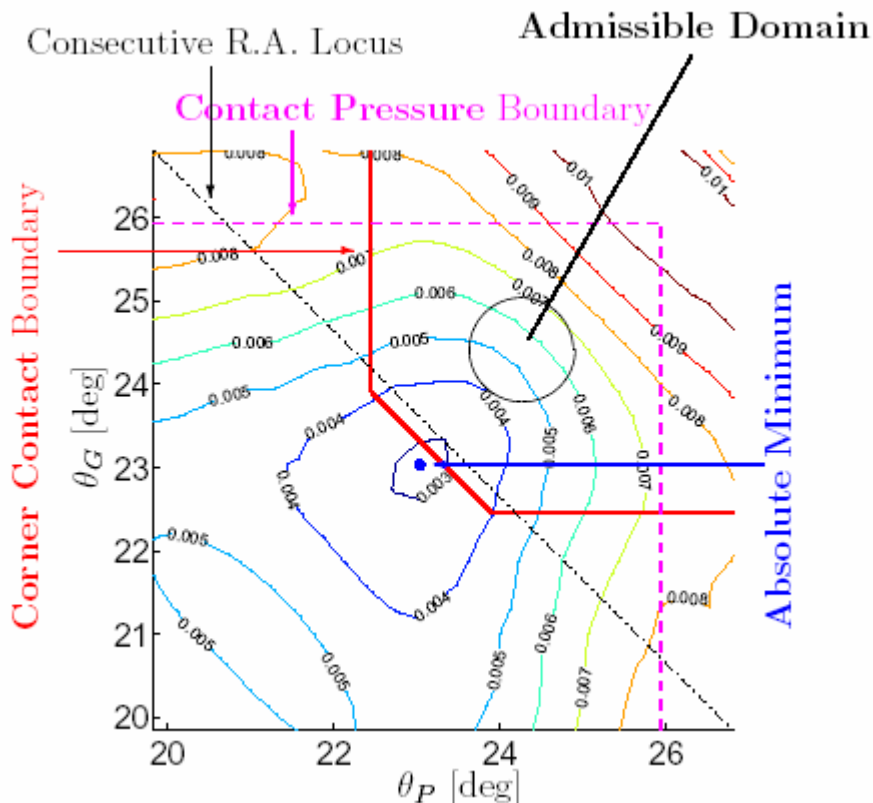


Figure 15. Application of boundaries on optimization for *LCR* gear set.



### 3.2 HCR spur gear set

The analyzed *HCR* gear set design parameters are summarized in table 3.

Pinion N. of teeth	38
Gear N. of teeth	47
P	0.3543 [mm <sup>-1</sup> ]
Pressure angle	20 [deg]

Table 3. HCR gear set parameters.

No symmetry was expected in this case as the pinion and gear are not equal. The total amount  $v_e$  was found with the previously discussed method (figure 3). The *PSTE* for the HCR gear set is shown in figure 16. No boundaries are presented yet. A very important feature about *PSTE* is that there is a zone in the domain  $(\theta_p, \theta_G)$  in which *PSTE* is almost constant.

It is also remarkable that for *HCR* no unique local minimum was found, but there are two of them, one at little  $(\theta_p, \theta_G)$  and the other at high  $(\theta_p, \theta_G)$ .

#### 3.2.1 Application of boundaries to optimum profile research

Taking into account the boundaries previously discussed, the results are shown in figure 17.

Both the local minima found are outside the admissible domain. Thus In HCR gear sets, the *Consecutive Start Relief Roll Angle* rule can not

be applied as this condition produces Corner Contact, if the total amount  $v_e$  is estimated in the same way discussed above for LCR gear sets. However for too large tip relief, the Contact Ratio can fall below 2 especially if the torque applied is little, and so the HCR behavior is missed.

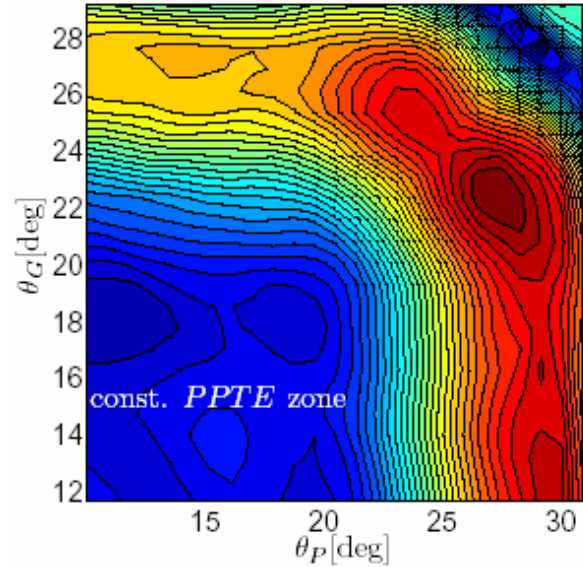


Figure 16. Evidence of *PSTE* constant zone for the HCR spur gear set.

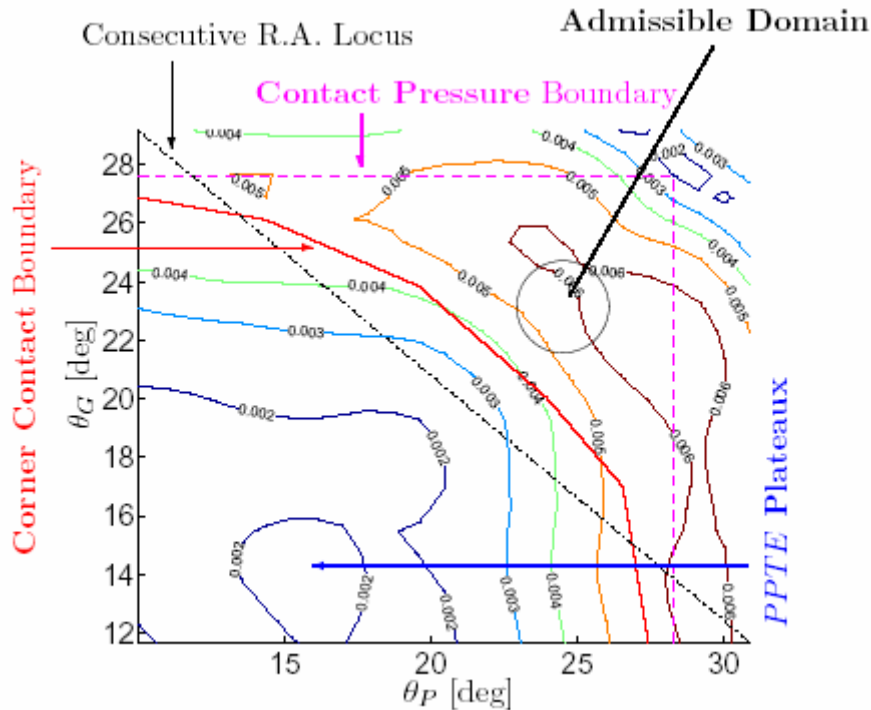


Figure 17. Application of Boundaries in HCR spur gear set.

But if tip relief  $v_e$  is properly increased (in this case by a factor 1.5) Contact Ratio does not reduce below 2 and Corner Contact boundary moves toward the left bottom corner in the  $(\theta_p, \theta_G)$  domain.

At this point the Consecutive Roll Angle rule can be applied again because the Consecutive Roll Angle Locus now falls inside the acceptable region of the domain (figure 18). Following the Consecutive Roll Angle Locus the minimum *PSTE* is found near the edge of the constant *PSTE* zone.

In figure 19 it is shown  $PSTE(\theta_G)$  along the Consecutive Start Relief Roll Angle Locus.

To show the strong interaction between the start profile modification roll angles and contact ratio, contact pressures is shown in figure 20 for different positions along the Consecutive Roll Angle Locus.

Pressure curves overlaps are directly related to contact ratio. It is worth noting that the minimum  $PSTE$  was found when contact ratio is exactly 2.

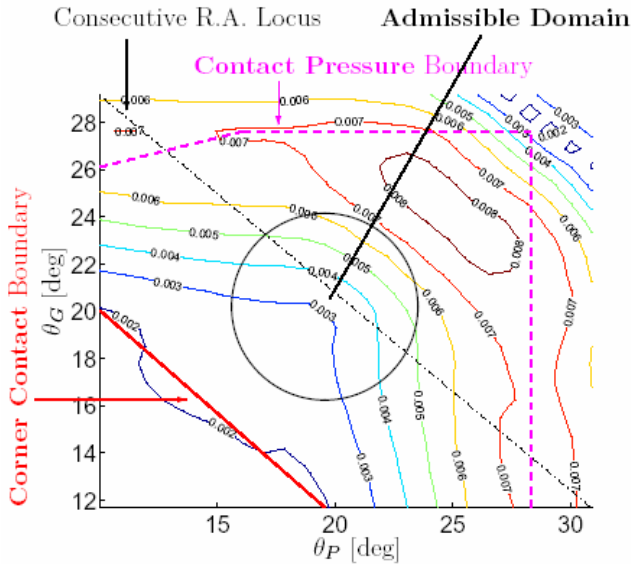


Figure 18. Boundaries with increased total relief amount  $v_e$ .

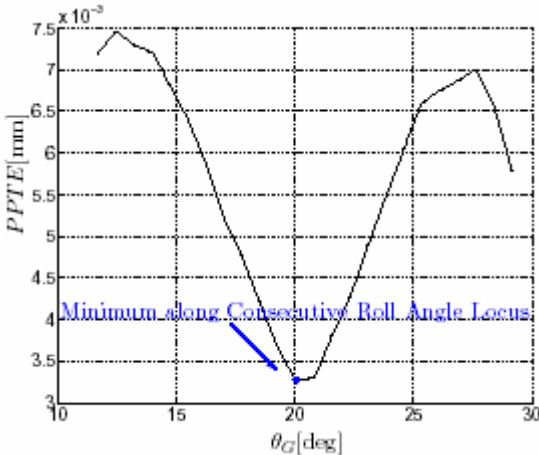


Figure 19. Minimum  $PSTE$  along Consecutive Roll Angle Locus.

## Conclusions

The aim of the work is to develop a simple method for profile optimization at nominal torque in terms of  $PSTE$  which is the main cause of *whining noise* in spur gears.

A combined semi-analytical and FEM software has been used for performing meshing simulations.

LCR and HCR spur gears have been studied separately. In LCR the suggested method can be summarized in the following steps:

1. The maximum tooth detection is calculated (without modification) and its value put equal to the total tip relief  $v_e$ .
2. Total tip relief is initially divided equally between Pinion relief and Gear relief then *Niemann* long profile modification with nominal torque is applied.
3. After analysis first run, pressure behavior along contact length is considered.

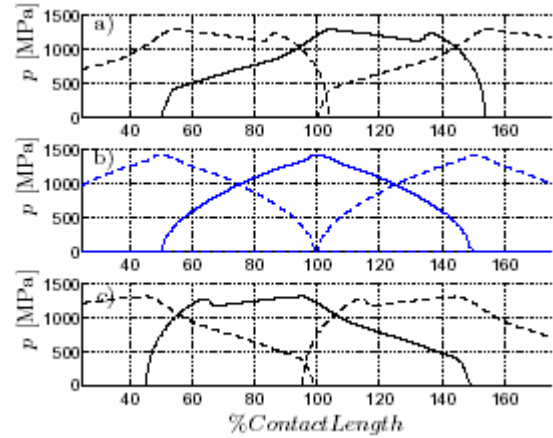


Figure 20. Contact Pressure along Consecutive Roll Angle Locus for different combinations  $(\theta_P, \theta_G)$  and Contact Ratio evidence. (a) At high  $\theta_P$  and low  $\theta_G$ . (b) At minimum  $PSTE$ . (c) At low  $\theta_P$  and high  $\theta_G$ .

4. If corner contact is detected, then tip relief amount for pinion and gear are increased according to the side at the corner contact is found.
5. Consecutive Start Relief Roll Angle are assumed, and then only one parameter ( $\theta_P$  or  $\theta_G$ ) is to consider for the optimization analysis.
6. The minimum  $PSTE$  in one variable can be found in few steps due to the fact that the minimum is unique.

For HCR spur gears, the steps above can be applied as well, but from points 1, 2, 3 and 4 of the procedure the estimated  $v_e$  is not high enough to get the *Consecutive Roll Angle Locus* inside the Admissible Domain. This is mainly due to the fact that the *Niemann* rule, which is the start point of the method, is valid only for *LCR* spur gears.

It is then necessary to properly increase Total Relief Amount  $v_e$ . Doing this corner contact is eliminated along *Consecutive Roll Angle Locus* but *Contact Ratio* could be lessened below 2.

A convenient trade-off is then needed.

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## References

- [1] J.D. Smith. Gears and their vibration, A Basic Approach to Understanding Gear Noise. The Macmillan Press LTD., 1983.
- [2] S. Harris. Dynamic loads on the teeth of spur gears. *ProclMechE*, 172(2):87, 1958.
- [3] R. G. Munro, N. Yildirim, and D.M. Hall. Optimum profile relief and transmission error in spur gears. *Proceedings of the First International Conference on Gearbox Noise and Vibration*, pages 35 - 41, 1990.15
- [4] G. Niemann and H. Winter. *Machinenelemente Band II, Getriebe allgemein, Zahn-radgetriebe*, volume 2. *Grundlagen, Stirnradgetriebe*, Springer-Verlag, Berlin, 1983.
- [5] G. Niemann and J. Baethge. Transmission error tooth stiffness, and noise of parallel axis gears. *VDIZ*, 112(4), 1970.
- [6] H. Walker. Gear tooth deflections and profile modifications. *Engineer*, 166: 409-412, 434-436, 1938.
- [7] S.M. Vijayakar and D.R. Houser. Contact analysis of gears using a combined finite element and surface integral method. *Proceedings of FIGMA Technical meeting*, 1991.
- [8] S.M. Vijayakar, D.R. Houser, and H.R. Busby. A summary of a new finite element technique with special application to gears. *Proceedings of Int. Conf. of Gearing*, 2:813 - 817, 1988.
- [9] M. S. Tavakoli and D.R. Houser. Optimum profile modification for the minimization of static transmission errors of spur gears. *Proceedings of ASME 84 - DET - 173*, 1986.
- [10] M. Beghini and C. Santus. Analysis of plane contact with discontinuous curvature. *International Journal of Mechanical Sciences*, 2004. In Press.
- [11] M. Beghini, G.M. Bragallini, F. Presicce and C. Santus. Influence of the linear tip relief modification in spur gears and experimental evidence. *ICEM 12*, 2004. Bari.
- [12] S.M. Vijayakar. *Calyx Users Manual*. Advanced Numerical Solutions, Hilliard OH, March 2003.
- [13] S.M. Vijayakar. *Helical3D User's Manual*. Advanced Numerical Solution, Hilliard OH, March 2003.