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INFLUENCE OF THE LINEAR TIP RELIEF MODIFICATION IN SPUR GEARS AND EXPERIMENTAL EVIDENCE

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ABSTRACT
In this paper issues about tip relief modification and its effect in terms of surface loading of the tooth flank are investigated. The linear tip relief modification is considered. This kind of modification is considered to be potentially dangerous for contact pressure which is the primary reason of micro-pitting activation. Indeed the nominal tip relief topography does not provide further continuity than only C^0 and then gives rise to a sharp edge. In the paper an effective profile is defined on the basis of direct measurements and more realistic contact pressure at start relief profile modification is determined. Numerical results are compared to experimental evidence.

1. INTRODUCTION

According to the rigid geometry in spur gears, the involute conjugate profiles produce regular motion between drive and driven gears, but even neglecting geometrical errors, loaded gear teeth deflection modifies the relative motion generating the Transmission Error.
In order to reduce Transmission Error, tip relief profile modification is usually introduced particularly for high loaded gears. The profile modification is effective also for avoiding the corner contact which can be a non-conjugate contact at the extremities of mesh cycle generated by the teeth deflection.
In the design, tip relief profile is defined as the thickness v of the material removed along the tooth flank with reference to the nominal involute profile. Profile modification is usually defined versus the Roll Angle coordinate (θ) and measured in the direction of the inner normal (Fig. 1).
According to the definition, the profile modification is completely defined by:

- relief topography (type of $v(\theta)$ function),
- the position P_s where the start relief begins and the related start Roll Angle (θ_s),
- the amount of relief at the end of the flank v_e.

In high performance spur gears, the tip relief profile modification assumes a key role in design [1, 2]. As observed, if high torque is applied to the gear pair, the tooth deflection can generate corner contact, due to increase of the true contact length, and whining noise related to the increase of Transmission Error. Several works have been carried out to find the optimum modification in particular for reducing the vibration [3, 4, 5,6].

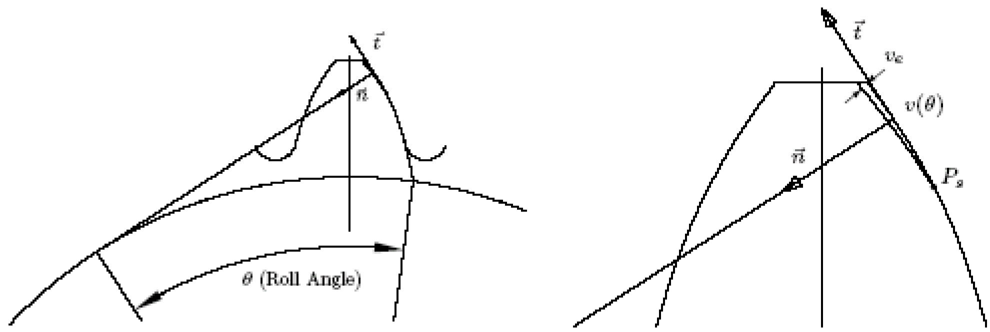


Fig.1. Definition of the profile modification: Roll Angle (θ), tangent and normal vectors $(\vec{t}, \vec{n})$, thickness of material removal v(θ), Start relief point P_s and total thickness at the end of the flank v_e.

In the present paper the linear relief topography is considered, which seems to be the best in terms of Transmission Error reduction [7] even though, as focused in the followings, it creates some problems regarding the contact pressure between the meshing gears.
In fact, if the nominally designed relief is considered, for the lack of tangency of the profile at P_s, the solution of the contact problem is expected to be singular, and the contact pressure locally unbounded, [4].
In Fig. 2 the condition in which contact region encompasses two start relief points (P_{s1}, P_{s2}) is shown. This situation is common as start relief roll angles are almost consecutive during meshing cycle because their distance is comparable with the width of the contact region.

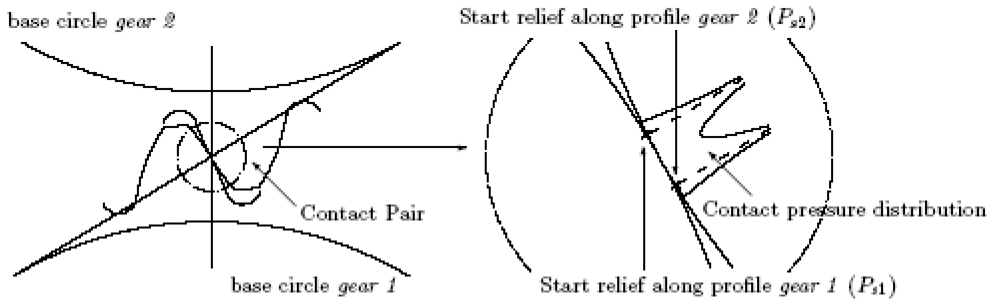


Fig.2. Schematic condition of singular contact pressure when the contact is near the P_s.

When the contact region does not encompass any start relief point, the relative curvature is well defined and it can be considered almost constant inside the contact area. In this case the contact pressure can be well approximated by the Hertzian relation. The weak perturbation due to the varying relative curvature inside the contact region is treated in [9].
However, several reasons can be considered for neglecting singular solutions for the contact pressure:

- every technological process can not produce a perfectly sharp edge,
- after a short run-in the material necessarily yields or wears, and sharp edges should be blunted,
- the angle at the tip relief start point is so flat that the surface roughness has to be considered for defining the corner region.

As a consequence, the macro-geometrical properties of the matching profiles have to be redefined in order to properly model the contact between the two elastic bodies.
Pitting (or micro-pitting) is the main type of damage related to high contact pressure, especially in rolling (little sliding) contact condition [10]. When the relative profile is regular (C^2 condition) the Hertz model is usually considered acceptable to predict the contact

pressure, while for C^1 contact condition, generalized models is needed [11]. In the present paper a method to evaluate the contact over-pressure in correspondence of the start tip relief is proposed.

2. NUMERICAL RESULTS

2.1 Contact pressure according to nominally profile

A gear set with modulus m = 1.75 mm, 80 - 80 teeth number is considered. LTCA (Loaded Tooth Contact Analysis) analysis is performed under static load and frictionless hypotheses.

To predict the contact pressure on tooth flank during meshing, numerical analyses have been conducted. A complete Finite Element Analysis (ANSYS™), and a hybrid semi-analytical finite element technique software, dedicated to the contact problem (described in [12]), has been used.

Without going into details of the mathematical techniques, it is interesting to focus that a numerical discretization is not able to reproduce a singular pressure distribution. Thus, to detect the singularity of the solution, a convergence analysis has to be developed, for instance by increasing nodal density and analyzing the trend. Thus, when contact region includes the P_s points, the numerically predicted pressure diverges as the local discretization is improved.

The maximum contact pressure vs. the meshing Roll Angle is reported in Fig. 3, for the gear set here considered. It can be verified that adopting the nominal profile a contact singularity is produced. Indeed when the edge of the flank is inside the contact region the contact pressure does not seem to be bounded, as the discretization is improved.

It can be also observed that the three models give equivalent results when the solution is not perturbed by the sharp edge contact, while the solution becomes mesh dependent otherwise.

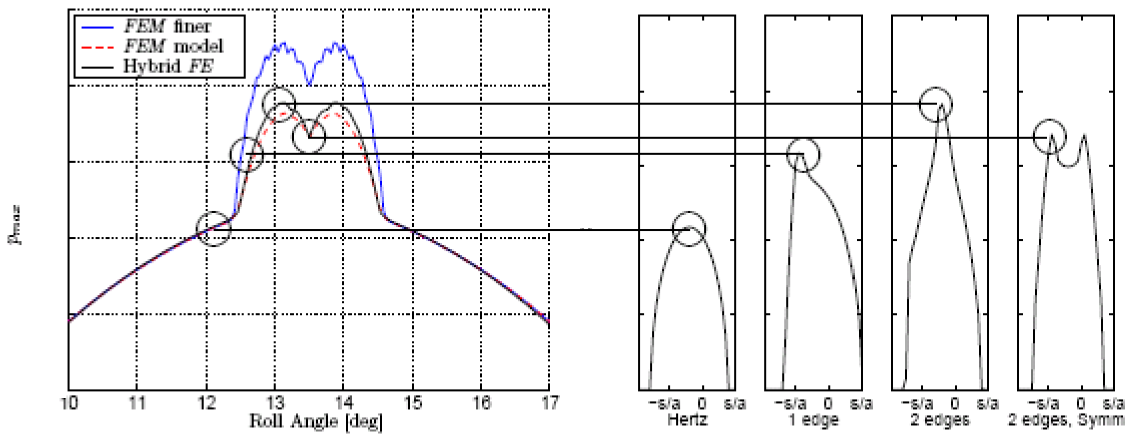


Fig.3. Example of different levels of discretization and their effects on contact pressure solution.

2.2 Definition of an effective regular profile

A regular profile reproducing the characteristic of the tip relief is proposed in order to avoid the singularity.

The basic idea is that a proper effective regular profile with superimposed roughness could be used to generate probabilistic distribution of profiles near P_s similar to that produced in regular macro-geometry profiles.

To obtain a reasonable effective profile the main guidelines were:

- the profile needs satisfying C^1 continuity,
- the profile in the zone of P_s has a curvature influenced by the level of roughness,
- the profile has to reproduce the measured profile as much as possible,

- the method must to be simple.

Several attempts have been tested to select the final procedure suitable to obtain the equivalent regular profile. Primarily the possibility to include in the model the micro-geometry by introducing a conforming intermediate layer between the two elastic solids was considered. In this models a properly defined layer reproduces the indirect contact through the roughness. The stiffness of the layer is to be considered strongly non linear and uncoupled, simulating the contacts of the asperities acting separately. This model was suggested by Greenwood-Williamson (GW) [13] and an experimental verification was given by Handzel-Powierza [14]. A good agreement was found at low contact pressure, while GW's model overestimates the contact stiffness where pressure is around the yield point mainly due to its linear hypothesis.

As in high performance spur gears, pressure is very high, this approach was not applied.

For this reason, solid elastic contact with properly chosen profiles was considered.

A high precision measurement machine was adopted in order to obtain the effective geometry of the tooth. A wide data base was prepared containing 800 profiles of 25 nominally equal gears (Fig. 4 (a)).

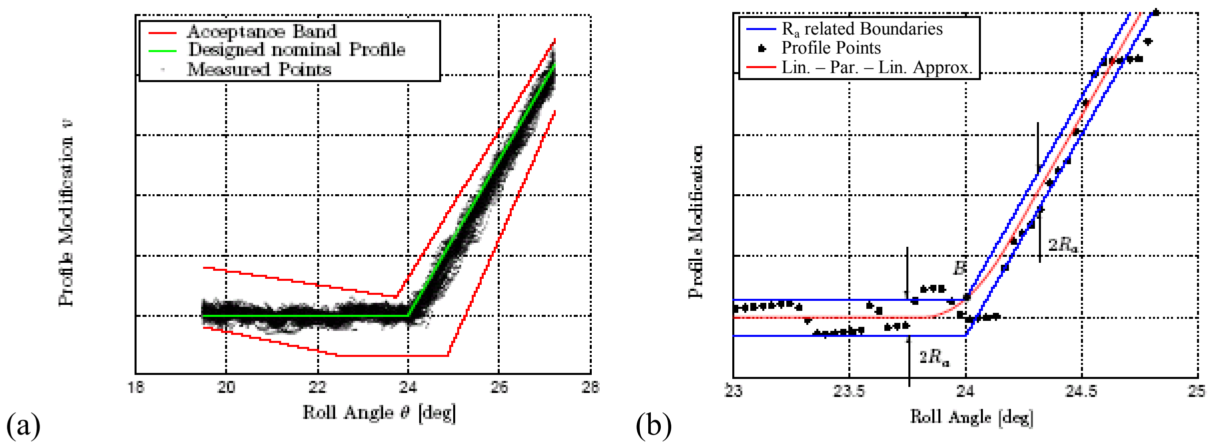


Fig.4. (a) 80 profile acquisition points (among the 800) with design tolerance band,
(b) Example of Linear-Parabolic-Linear approximation inside Ra related boundaries.

The first attempt to find the equivalent profile was spline least square fitting. Cubic spline was used with either C^1 or C^2 continuity. Equal length imposed segments or alternatively self proportional segments techniques were implemented. However, several problems arose:

- results in terms of contact pressure were strongly sensitive to parameters' tuning,
- profiles could be distorted even far from P_s where the nominal geometry is reliable,
- no direct control of the local curvature could be obtained.

In order to obtain more regular effective profile, a simpler topography was considered for the linear tip relief: segment - parabolic fillet - segment. The only parameter to set is the parabolic fillet curvature which was related to roughness of the profile (for example the average roughness R_a).

The leading idea was that R_a represents the width of the band where the major part of the points of the real profile is contained. Then the effective profile was assumed to be the smoothest curve inside that band.

According to this criterion the parabolic fillet has the largest curvature when it passes tangent to the band corner B (Fig. 4 (b)).

In figure 5 it is shown how the modified involute curvature is lower at the linear tip relief modification than the involute profile, but quite stronger at the parabolic fillet.

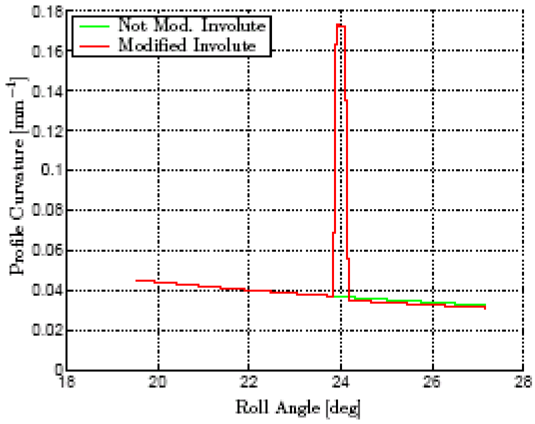


Fig.5. Comparison between Not modified and Modified profile curvature.

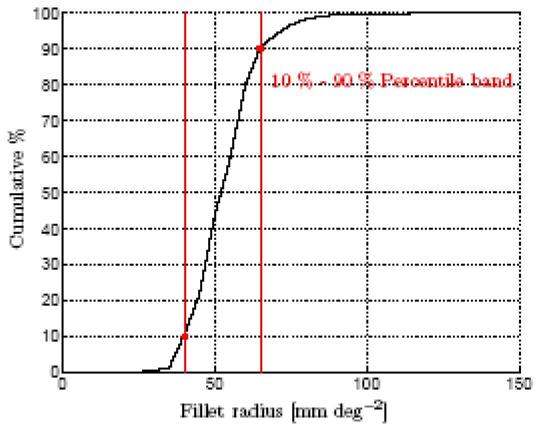


Fig.6. Statistical evaluations, cumulative distribution.

This approximation has been produced for each of the 800 profiles in the data base and statistically evaluated.
Distribution and 10 % - 90 % band of fillet radius population were found (Fig. 6).
Since the fillet radius range for the 10%-90% band is significantly narrow with respect to the mean curvature value, this simple profile approximation related to the roughness R_a parameter could be considered representative of the local tooth surface.
It is worth noting that the fillet curvature is reasonably underestimated since the largest fillet inside the band is chosen. As a consequence the related contact pressure pattern is expected to be a reasonable underestimation of the local value.

2.3 Meshing simulations with effective regular profiles

To evaluate the whole range of possible profiles in contact, simulations were performed putting into contact mean curvature value, of the statistical distribution, with mean curvature value (symmetric graph of Fig. 7) and 10 % value with 90 % value of curvature cumulative distribution (not symmetric graph of Fig. 7).

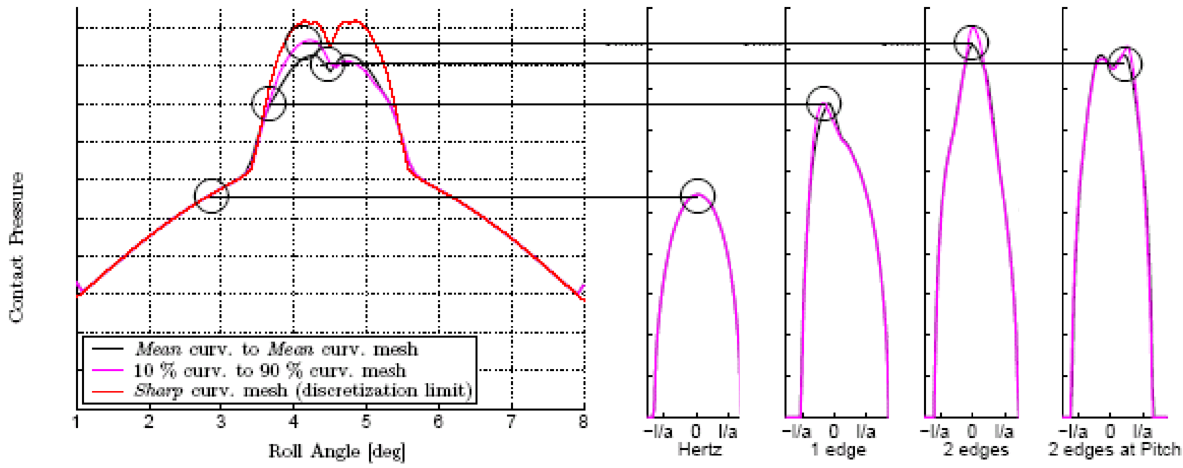


Fig.7. Contact pressure mesh analysis with combinations of fillet approximation profiles: maximum contact pressure and contact pressure patterns.

As the curvature variation is limited (figure 6), similar contact pressure levels were found for any analyzed combination.

It is worth noting that even with effective regular profiles, contact pressure patterns are similar to those shown in Fig 3. This is due to the fact that the parabolic fillet is a small zone of contact region.
As a consequence the point of maximum pressure travels along a larger segment of the mating profile. According to this the effect of high contact pressure is reasonably expected to be found in a portion of each profile wider than the fillet region.

3. EXPERIMENTAL EVIDENCE

As evidence of the critical conditions experienced by the material, near the start tip modification, an experimental result is here provided, obtained on an AVIO high speed power circulating gear test rig running at C.R.T.M. of the University of Pisa.
After a short run in, the surfaces in contact were analyzed by means of a profilometer.
In order to have the minimum slope of the tooth edge, an optimum displacement angle β was obtained through a high precision step motor (Fig. 8).

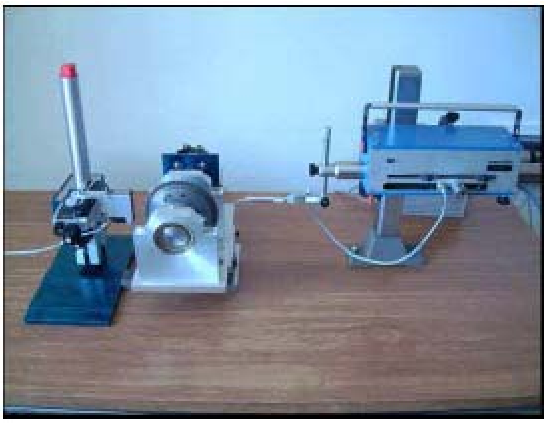
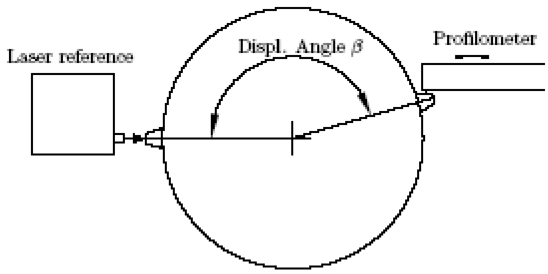
(a)  (b) 

Fig.8. Displacement equipment for roughness acquisition: (a) equipment photograph, (b) scheme.

A very accurate acquisition was performed (10 hours long) and a high definition surface map obtained. The portion of investigated surface was 3.0 mm x 3.0 mm. As in the sliding direction, the profile curvature produces a distortion of length, numerical correction was introduced in order to represent acquisition points in Roll Angle coordinate on the basis of the β angle.
To estimate the position on the tooth surface, the tooth edge reference was assumed. As the tooth edge is rounded, the position of the active profile end point was estimated with an accuracy of ± 25 μm.
In Fig. 9 the numerical simulated contact pressure are reported in comparison with the experimentally obtained surface map.
The micro-pitting zone is pointed out. It can be observed that the region of high contact pressure along the profile is very close to the region of the surface where typical micropitting craters were detected.
It is worth noting that this evidence cannot be considered as a demonstration of the direct effect of the local geometry on the surface damage. Indeed, many other parameters, not included in the present analysis, contribute to the surface damage. Nevertheless, it cannot be excluded that the local high curvature induced by the linear tip relief can strongly contribute to this phenomenon.

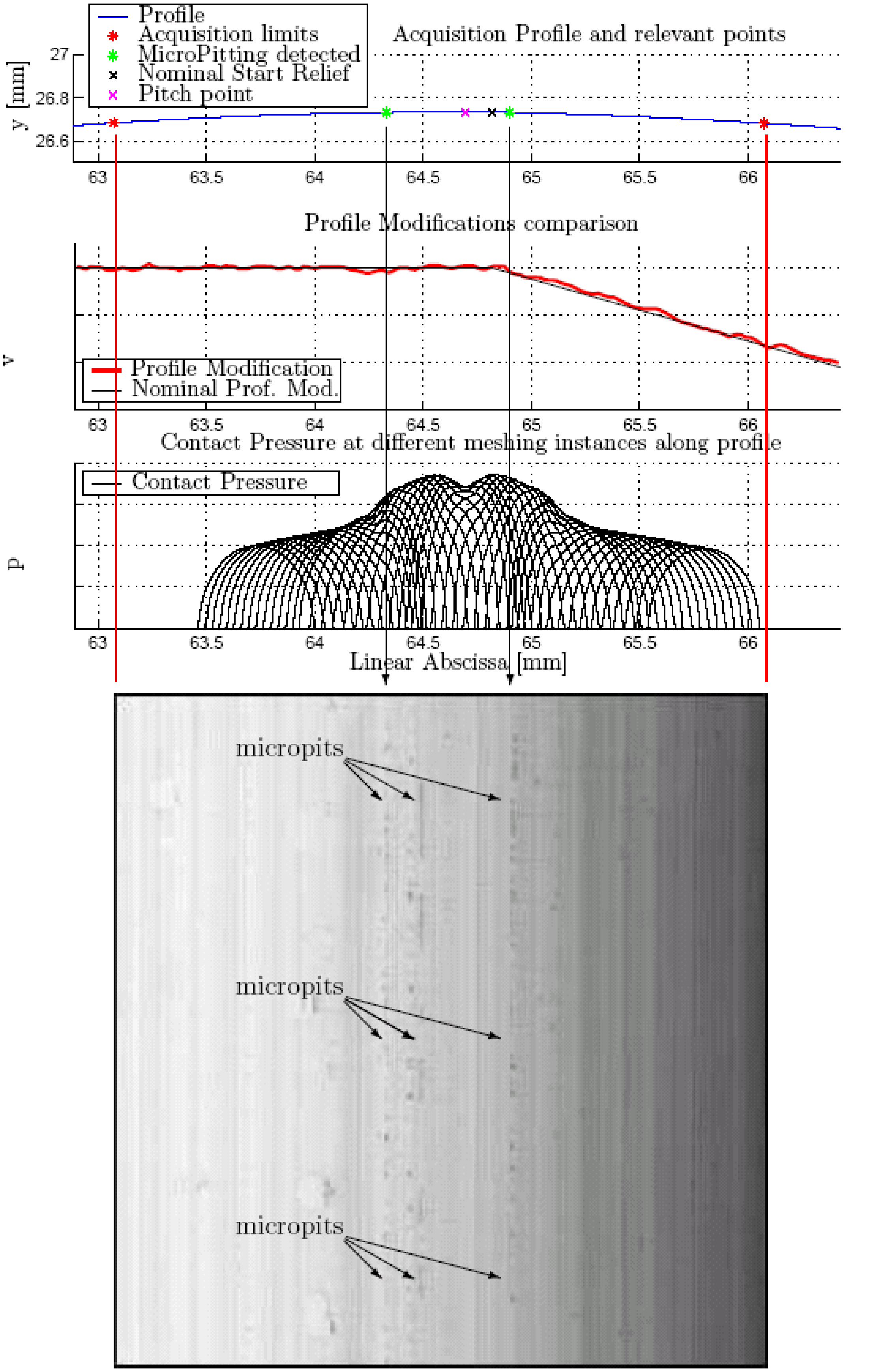


Fig.9. Comparison between profile acquisition and numerical results. Figures have the same abscissa scale

4. CONCLUSION

A simple method for defining an equivalent profile, in the case of linear tip relief, was shown. An underestimation of the curvature at the start of the profile relief, that is the critical geometrical parameter for the local contact pressure, was defined on the basis of the roughness parameter R_a.

After a statistical analysis of measured profiles, numerical simulations were performed, showing the presence of a region near the start of the tip relief where the contact pressure is significantly higher than in the other positions of the profile. In the same region, micro-pitting was observed suggesting that the local peak of maximum pressure could contribute to the surface damage.

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