Proposal for Tip Relief Modification to reduce Noise in Spur Gears and sensitivity to meshing conditions

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Abstract
In the paper a new Tip Relief Profile Modification for spur Gears is presented. The topography here proposed is a classical linear profile modification with a parabolic fillet (Linear-Parabolic modification).
The parabolic fillet extension is treated as a parameter and its effect investigated.
The proposed topography is able to take advantages from both Linear, and Parabolic tip relief. The evolution from Linear to Parabolic is discussed in terms of Peak to Peak Transmission Error. Anomalous contact conditions due to particular tip relief modification are reported, and maps are obtained in order to find optimum tip relief by taking into account contact conditions boundaries too.

Keywords: Transmission Error. Tip Relief Profile Modification. Spur Gears.

1. Introduction

In a gear set, Transmission Error (*TE*) is defined as the difference between the effective and the ideal position of the output shaft with reference to the input shaft. The ideal position represents a condition of perfect meshing, without geometrical errors or distortions. *TE* can be expressed either by an angular displacement or, more conveniently, as a linear displacement measured along the line of action tangent to the base circle. *TE* is considered to be the primary cause of whining noise [1]. Indeed, whining noise is produced by the change of tooth load:

- amplitude,
- position along the profile,
- direction.

These changing are consequences of tooth deflection, local contact deformation and body deformation, which are the origin of *TE*.
Several authors [2-6] studied the correlation between *TE* and Tip Relief Profile Modification, since it is a strong tool to modify *TE* by taking other parameters of the gear fixed. Niemann proposed long and short modifications [4]. This different denomination is based on the start point of tip relief along the profile. According to experimental results, gears with long modification show reduced *TE* excursion, indicated as Peak to Peak Transmission Error (*PPTE*), and therefore little noise level at the design torque. At lower torque this optimum condition was not verified and an intermediate or short modification is suggested.

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In tooth modification design, Tip Relief is defined as the thickness v of the material removed along the tooth flank with reference to the nominal involute profile. Profile modification is usually defined versus the Roll Angle coordinate (θ), Fig.1(a,b), and measured in the direction of the inner normal, Fig.1(c).

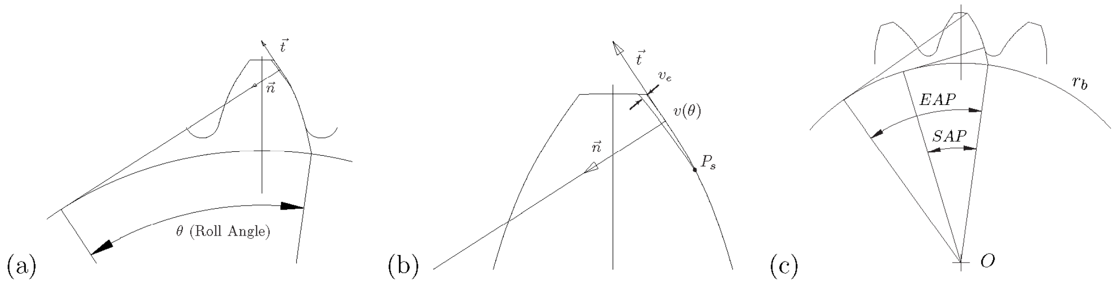


Fig. 1: (a,b) Definition of the profile modification function $v = v(\theta)$: Roll Angle (θ), Thickness of material removal v(θ), Start relief point P_s, and total thickness at the end of the flank v^e. (c) Definitions of: Start Active Profile (*SAP*), End Active Profile *EAP*.

The kind of function $v = v(\theta)$ is usually indicated as Profile Modification Topography. Meshing sensitivity to topography is the topic of the paper.

A different shape for the Tip Relief Profile Modification is proposed with the aim of reducing noise since *TE* can be significantly smoothed if an optimized profile modification is produced [1,5,3]. Litvin [7] agree with this approach even though Tooth Contact Analysis *TCA* is considered only, instead of Loaded Tooth Contact Analysis *LTCA*. The paper shows how the new profile tip relief modification here proposed can influence *TE* meshing response, according to *LTCA* hypothesis.

Linear and *Parabolic* topographies are the only shapes deeply studied [8-11].
Considering the recent grinding developments [12], it is possible to consider more complex shapes, in particular for spur gears in which profile modification is strategic.
Linear tip relief modification is of interest since it can produce little Transmission Error in spur gears if properly designed [4,11] at a given load.
Parabolic profile modification satisfies tangent continuity condition (even though sharp curvature change is produced, C^1 continuity) at the profile modification start point, while linear modification does not provide further continuity than only C^0 and then gives rise to a sharp edge. Indeed, according to solid elastic contact mechanics, sharp edge generates singular pressure when angular point falls inside the contact region [13], therefore this modification topography is considered to be more dangerous for contact pressure which is the primary reason of micro-pitting activation [14].

The new topography here proposed is able to exploit both (linear and parabolic) topographies properties, since it is a linear relief with parabolic a fillet, Fig.2.
The topography here showed was never found by the authors in the technical literature.
Transmission Error produced by the modification pattern of Fig.2 is showed in Fig.3 along with Fast Fourier Transform of *TE*. In Fig.3(b) mean value of the *TE* signal is eliminated, first frequency component is predominant and it is remarkable that with higher load *PPTE*, and first frequency amplitude, can smaller than with lower load.

Anyway, as discussed in [8], simply *PPTE* can be effectively considered as main parameter of *TE* roughness.

The tool here exploited to find optimum Tip Relief configuration is plotting a map showing *PPTE* as function of start relief Roll Angles θ_P , θ_G for fixed profile modification magnitudes at the top v^e_P, v^e_G.

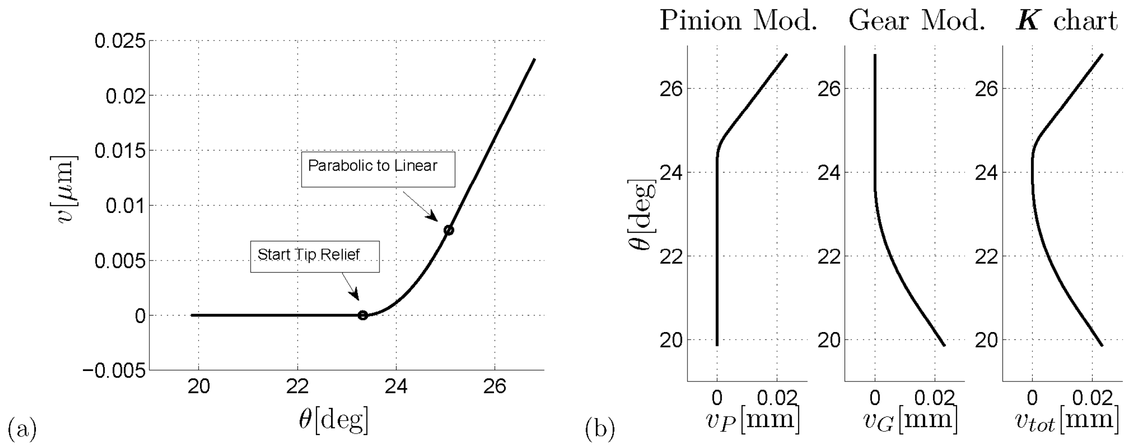


Fig. 2: (a) Tip relief profile modification definition. (b) Example of Pinion-Gear tip relief modifications and K-chart combination.

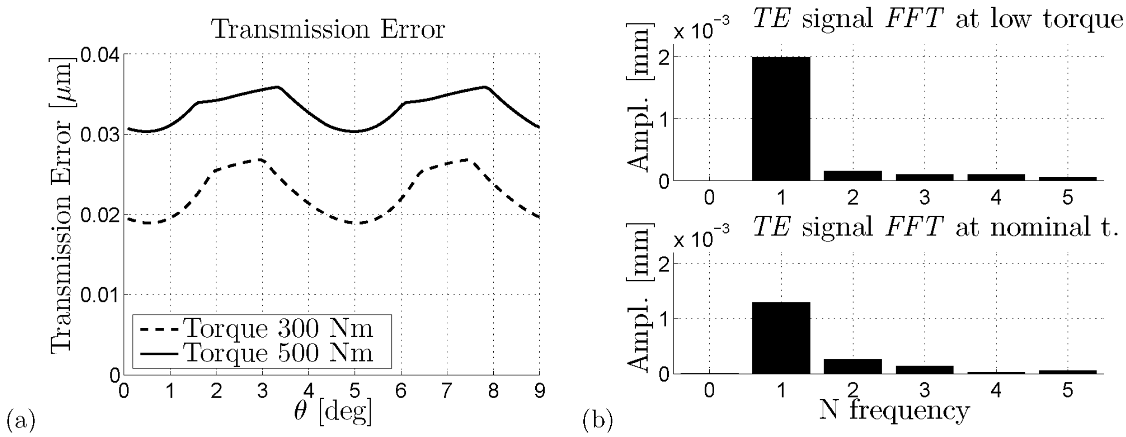


Fig. 3: (a) TE at low torque and at nominal torque applied. (b) Fast Fourier Transform (FFT) of the TE signal, both cases.

2. Methodology

The methodology proposed by the authors is here applied [8].

Meshing gears simulations have been carried out by means of an hybrid method, combining the Finite Element technique with a semi-analytical solution [15,16].

The main assumptions for the analysis are the following:

Plain strain conditions Suggested by the spur gear geometry (high ratio b/h). 2D plane strain analysis is adequate for this kind of tooth. Moreover the bi-dimensional version of the software requires little time both for model generation and simulations, with very high precision of results.

Static analysis Static *TE* was determined neglecting rotational speed and inertia forces. This is the main assumption, obviously undertaking dynamic analysis its too time consuming.

Friction neglected Since it is assumed that it has little effect on *TE* output.

Space error and pitch error not considered no statistical consideration was included in the analysis.

The quantities v^e_P (v^e_G) and θ_P (θ_G) are defined in Fig.1(a). The ranges for both these two variables are the Start Active Profile roll angle (*SAP*) and the End Active Profile roll angle (*EAP*) for each gear, Fig.1(b).

2.1. Research boundaries

***Corner Contact* boundary** Corner contact is produced when the contact region includes zones of the fillet of the tooth tip [17]. As a consequence of the teeth deflection the effective contact ratio is greater than that found according to rigid geometry hypothesis the contact pressure rises locally at the tip fillet, as in Fig.4.

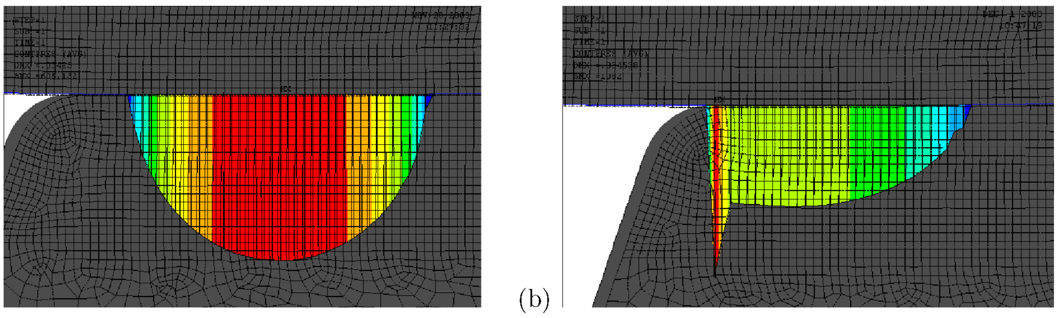
(a)  (b)

Fig. 4: (a) No contact corner detected. (b) Contact corner detected.

This definition of Corner Contact can be exploited if a FEM analysis is performed.
When the corner contact is detected, the calculated pressure peak was not considered reliable since the maximum is strongly affected by the radius of the fillet, which is a very unpredictable quantity.

***High Curvature* boundary** There is also the possibility to get anomalous contact condition if the tip relief is too high along the tooth profile in this condition High Curvature occurs and then contact pressure is expected to be much higher then Hertz model according to the nominal involute curvature.

In Fig.5 three contact pressure history are showed in which only Linear-Parabolic profile modification parameters are modified.
For each case two load conditions (*Nominal* load and *Low* load) are showed in comparison. It is worth noting that corner contact is related to Torque vs. Tip Relief, so configuration can show

corner contact if the load is high enough. This is obviously detectable only if Loaded Tooth Contact Analysis (*LTCA*) is considered

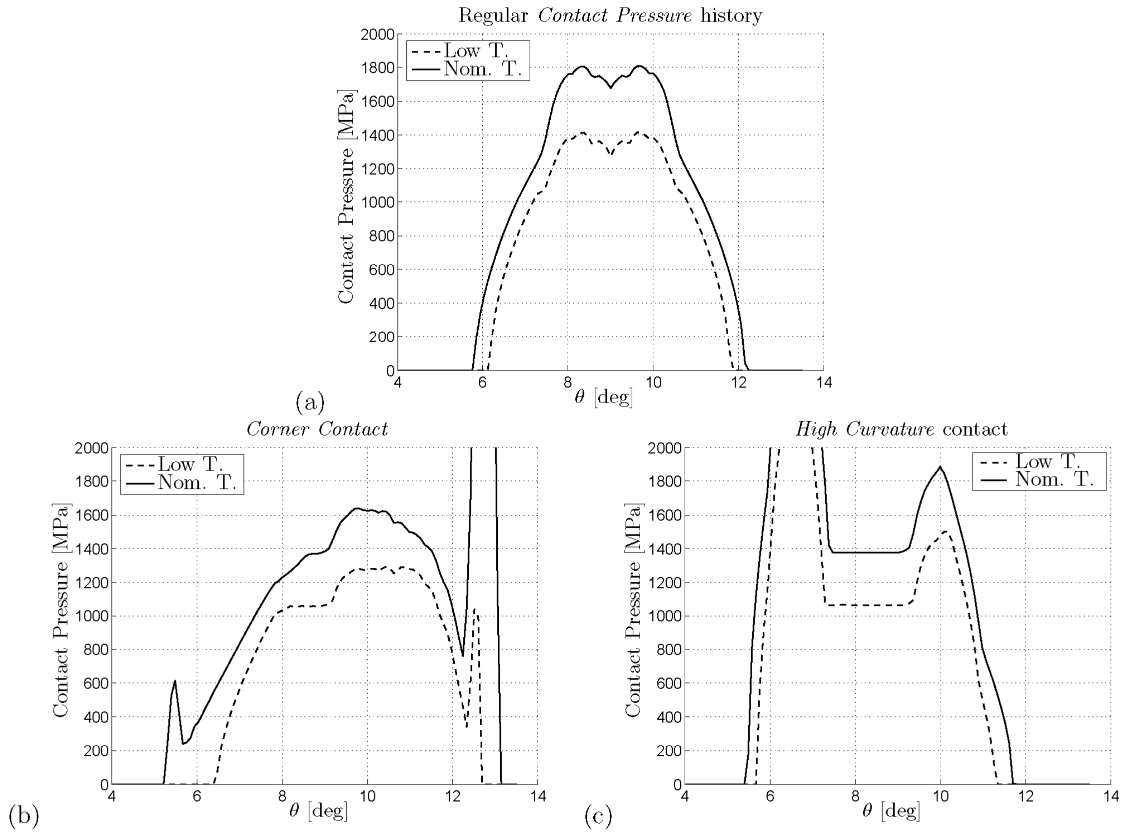


Fig. 5: (a) Regular contact pressure history configuration. (b) Corner Contact configuration. (c) High Curvature configuration.

2.2. *PPTE* maps

The paper main result is to obtain *PPTE* maps in order to find the minimum inside an acceptance domain defined according to Contact Pressure. Maps hereafter presented plot *PPTE* as function of Tip Relief Start Roll Angle (θ_P, θ_G), which are *PPTE* strong dependent parameters, for given quantity of material removed at the top ($v^e{}_P$, $v^e{}_G$). Initial values $v^e{}_P$, $v^e{}_G$ have to be related to the nominal torque of the gear set [8-11] even though they are strong dependent parameters as well, in terms of *PPTE*.
To define completely the Linear-Parabolic modification (θ_P, θ_G) and ($v^e{}_P$, $v^e{}_G$) are not enough, transition parabolic to linear point Roll Angles need to be defined (θ_{pP}, θ_{pG}), Fig.2(a).
It is clear that:

$$\theta_P < \theta_{pP} < EAP_P$$
$$\theta_G < \theta_{pG} < EAP_G \qquad (1)$$

To let transition from Linear to Parabolic be described in natural fashion, configurations at fixed ratios

$$\lambda_P = (\theta_{pP} - \theta_P)/(EAP_P - \theta_P)$$
$$\lambda_G = (\theta_{pG} - \theta_G)/(EAP_G - \theta_G) \quad (2)$$

are considered in any singular maps.
Limit configurations are $\lambda_{P(G)} = 0$ Linear modification and $\lambda_{P(G)} = 1$ Parabolic modification. Maps with λ ranging from 0 up to 1 are reported hereafter.

2.3. Computational performances
To perform parameters sensitivity analysis, a common PC platform was used with the following characteristics

- CPU 2.6 GHz
- RAM 1 GB

Plane strain analysis was performed by ExtPair2D™, [18, 19].
Analysis were automatically performed in about 4.5 CPU hours, simulating 50 time steps for each meshing, for 225 different tip relief (θ_P, θ_G) configurations.

3. Results
The above mentioned methodology was applied to a Low Contact Ratio (*LCR*) gear set.
Main parameters of the set are reported in Tab.1.

Table 1: *LCR* gear set design parameters.

Modulus	1.75 mm	Pressure angle	22.5 deg
Pinion N. teeth	80	Gear N. teeth	1.75 mm
Pinion external diameter	143.2 mm	Gear external diameter	80
Pinion root diameter	135.3 mm	Gear root diameter	143.2 mm
Pinion face width	11.0 mm	Gear face width	135.3 mm
Pinion v^e_P	23.3 μm	Gear v^e_G	11.0 mm

For the equal number of the set several symmetry properties are expected.
Configuration analyzed are reported in Tab.2

Table 2: Profile modification ratios, and configurations analyzed.

$\lambda_{P(G)}$	0	1/4	2/4	3/4	1
0	(Lin)				
1/4		(1-1)	(1-2)	(1-3)	
2/4		(2-1)	(2-2)	(2-3)	
3/4		(3-1)	(3-2)	(3-3)	
1					(Par)

Two load conditions applied to the gear set, were considered in the present paper:

- 300 Nm which is considered a low load for the gear set,
- 500 Nm which is the nominal load for the gear set.

3.1. Nominal load

For nominal torque the effect on *PPTE* of the migration from Linear to Parabolic topography is depicted in Fig.6.

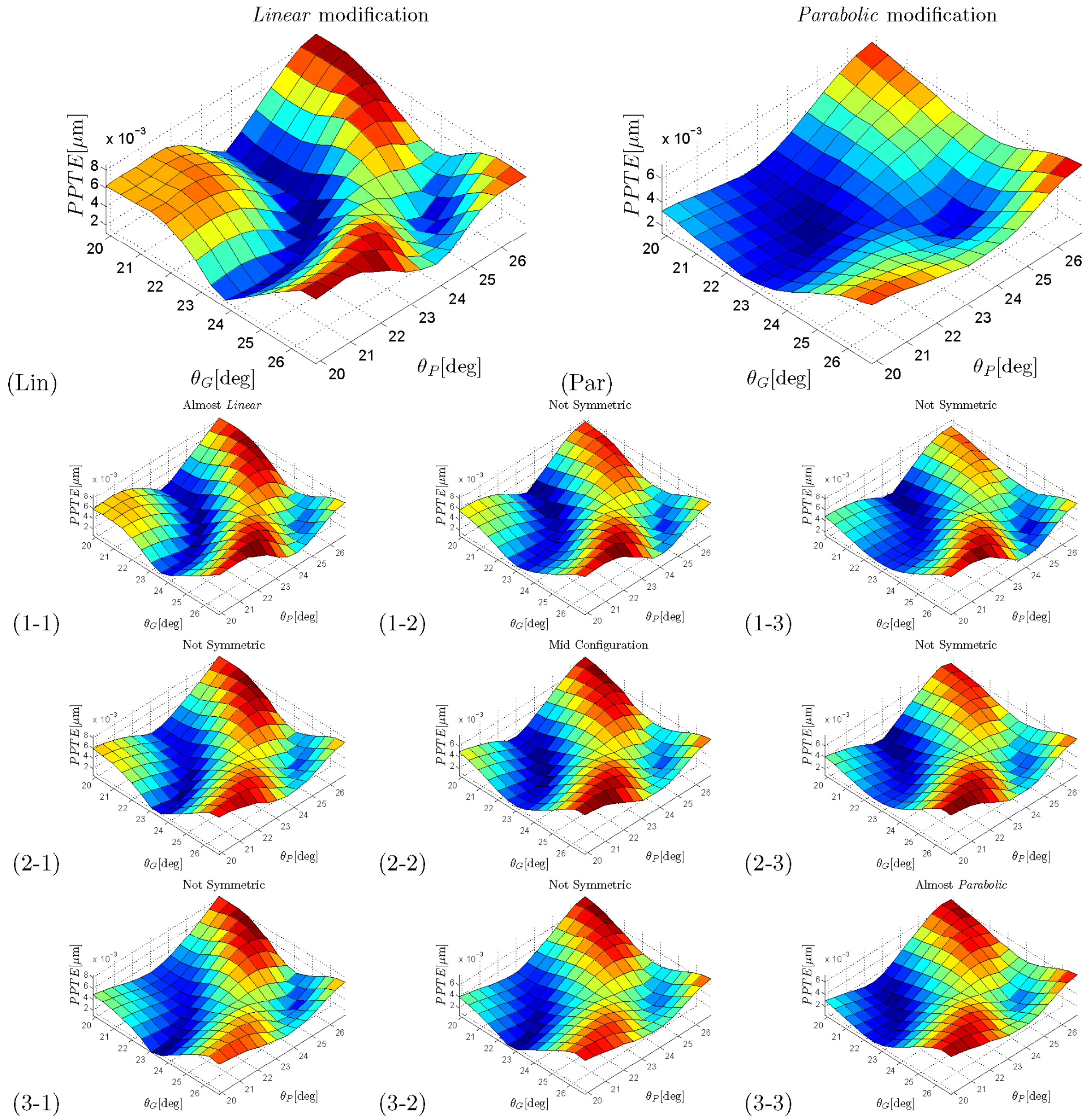


Fig. 6: Nominal load: (Lin) *PPTE* obtained with Linear modification topography. (Par) *PPTE* obtained with Parabolic modification topography. (*i-j*) Intermediate configuration meshing conditions.

Furthermore boundaries were applied. Results are reported, only for symmetric configurations, in Fig.7.

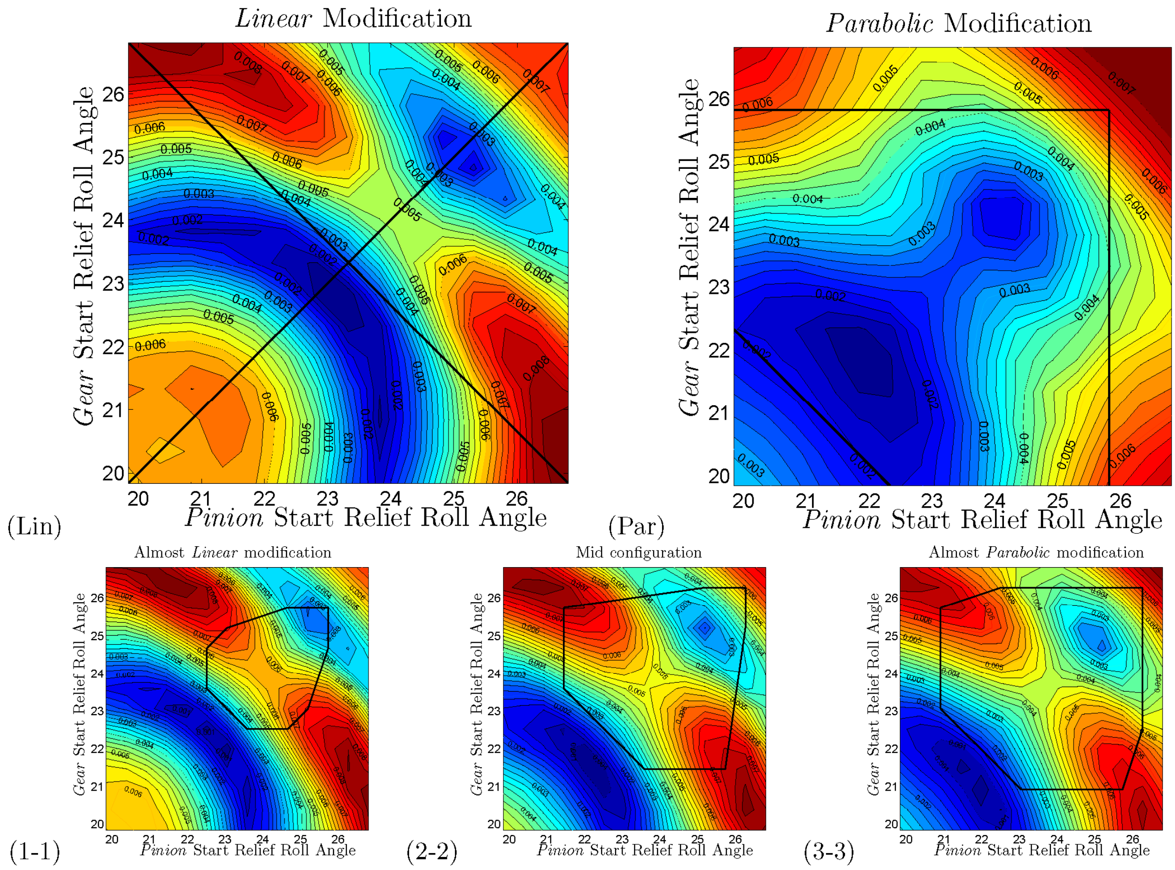


Fig. 7: Nominal load: (Lin) Linear modification, no boundary can be consistently applied. (Par) Parabolic modification, boundaries are wider. (*i*-*i*) Intermediate configurations.

When searching the minimum inside boundary, it is clear that Parabolic modification is the one that offers wider acceptance domain. Therefore larger area in which *PPTE* produces an absolute minimum can be considered.
Minimum values for configurations showed in Fig.7 are reported in Tab.3, not considering boundaries.

Table 3: Minimum PPTE values at the nominal load.

(Lin) 1.14 μm		
(1-1) 0.63 μm	(2-2) 0.76 μm	(3-3) 0.57 μm
(Par) 1.39 μm		

Among minimum values reported in Tab.3 only the one referred to Parabolic modification is the acceptable. Though others are lower, they fall outside the boundary. Taking into account boundary, parabolic modification leads to the best result.
In Fig.8 Transmission Error trace is reported both for Parabolic optimum (continuous line) and for Mid (2-2) minimum *PPTE* configuration.

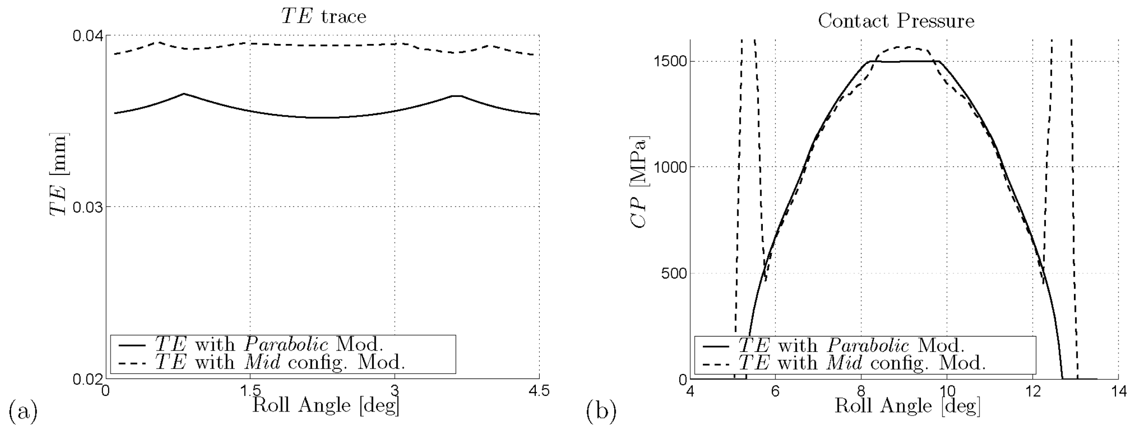


Fig. 8: (a) Parabolic optimum *TE* trace. (b) Mid configuration Linear with Parabolic fillet optimum *TE* trace.

PPTE is lower for Mid configuration, but Edge Contact is evident, indeed this configuration falls outside acceptance boundary as showed in Fig.7(2-2).

3.2. Low load

For low torque the migration from Linear to Parabolic topography is depicted in Fig.9.
For this load level no boundary related to contact pressure was applied, since load is not expected to be dangerous at this level. Even tough there necessary are conditions of anomalous contact (Edge Contact or High Curvature) also at lower load, it is possible to assume that they are confined to not useful configurations. Minimum values found at the different configurations are reported in Tab.4.

Table 4: Minimum PPTE values at the low load.

(Lin) 0.87 µm		
(1-1) 0.39 µm	(2-2) 0.53 µm	(3-3) 0.52 µm
(Par) 0.95 µm		

It is worth noting that nor Linear neither Parabolic topography generates minimum *PPTE* at both loads.
Minimum *PPTE* output is generated by a Linear to Parabolic configuration depending on the load. To propose this as “*golden*” rule is not the aim of the paper, since this can be strongly deviated by meshing parameter. The contribute of the paper is to show that the parabolic fillet can be an effective parameter to obtain an optimum solution.

3.3. Sensitivity to centre distance

Sensitivity to meshing conditions are here proposed.
In order to avoid a huge amount of graphical output, Linear modification at Nominal load sensitivity to centre distance offset is considered. Results are showed in Fig.10.
The entities of offset are around ±1/10 in comparison with modulus, it is clear to understand that perturbation induced by meshing condition have influenced little *PPTE* map.

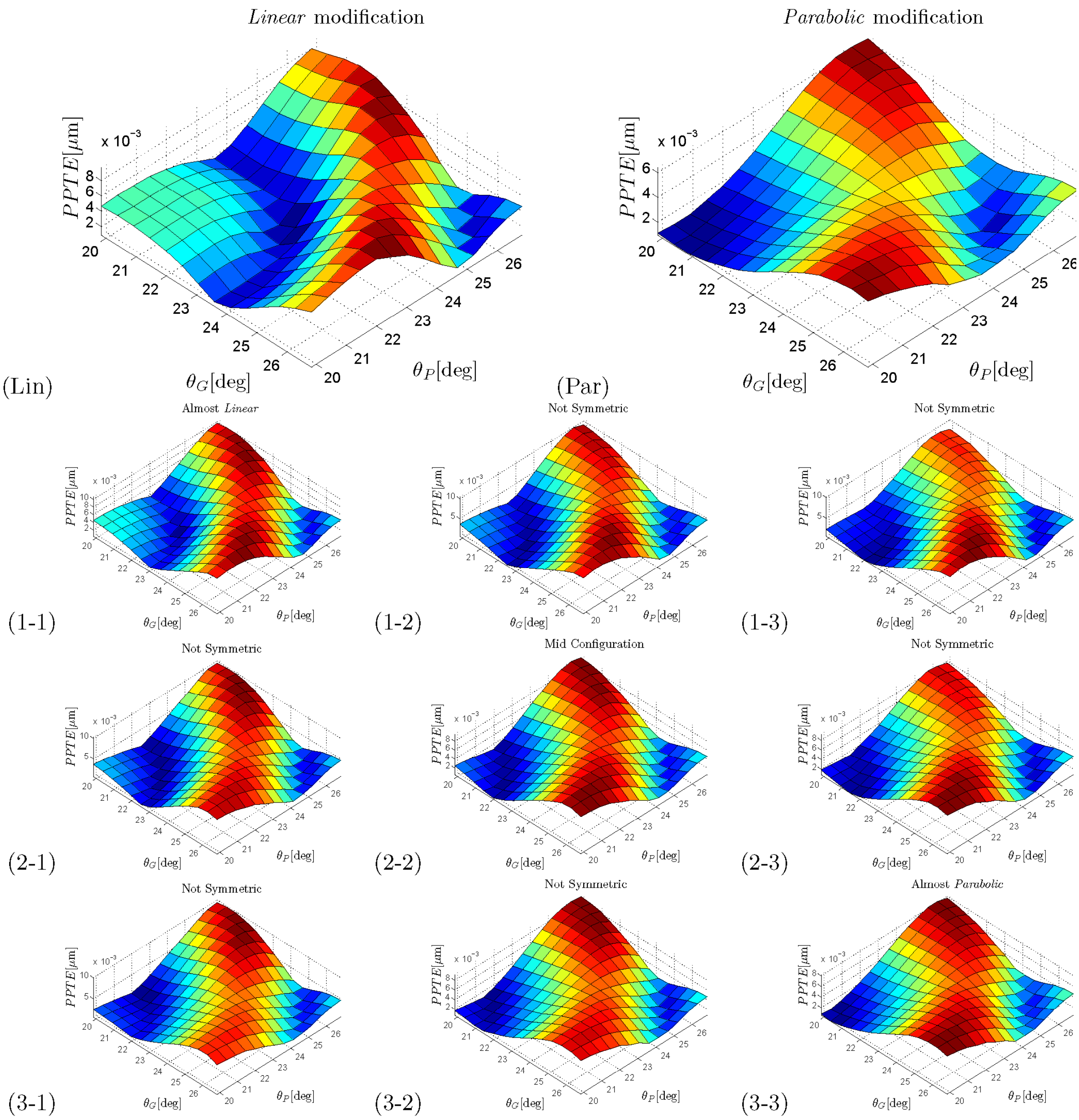


Fig. 9: Low load: (Lin) *PPTE* obtained with Linear modification topography. (Par) *PPTE* obtained with Parabolic modification topography. (*i-j*) Intermediate configuration meshing conditions.

4. Conclusions

In the present paper a new Tip Relief Profile Modification is proposed along with Loaded Tooth Contact Analysis methodology analysis. Static Peak to Peak Transmission Error (*PPTE*) is considered as main meshing output since it can be related to noise level. Extensive parametric numerical analysis results are presented. *PPTE* maps are reported as function of Pinion and Gear Start Tip Relief Roll Angles. Migration from Linear to Parabolic modification is effectively described.

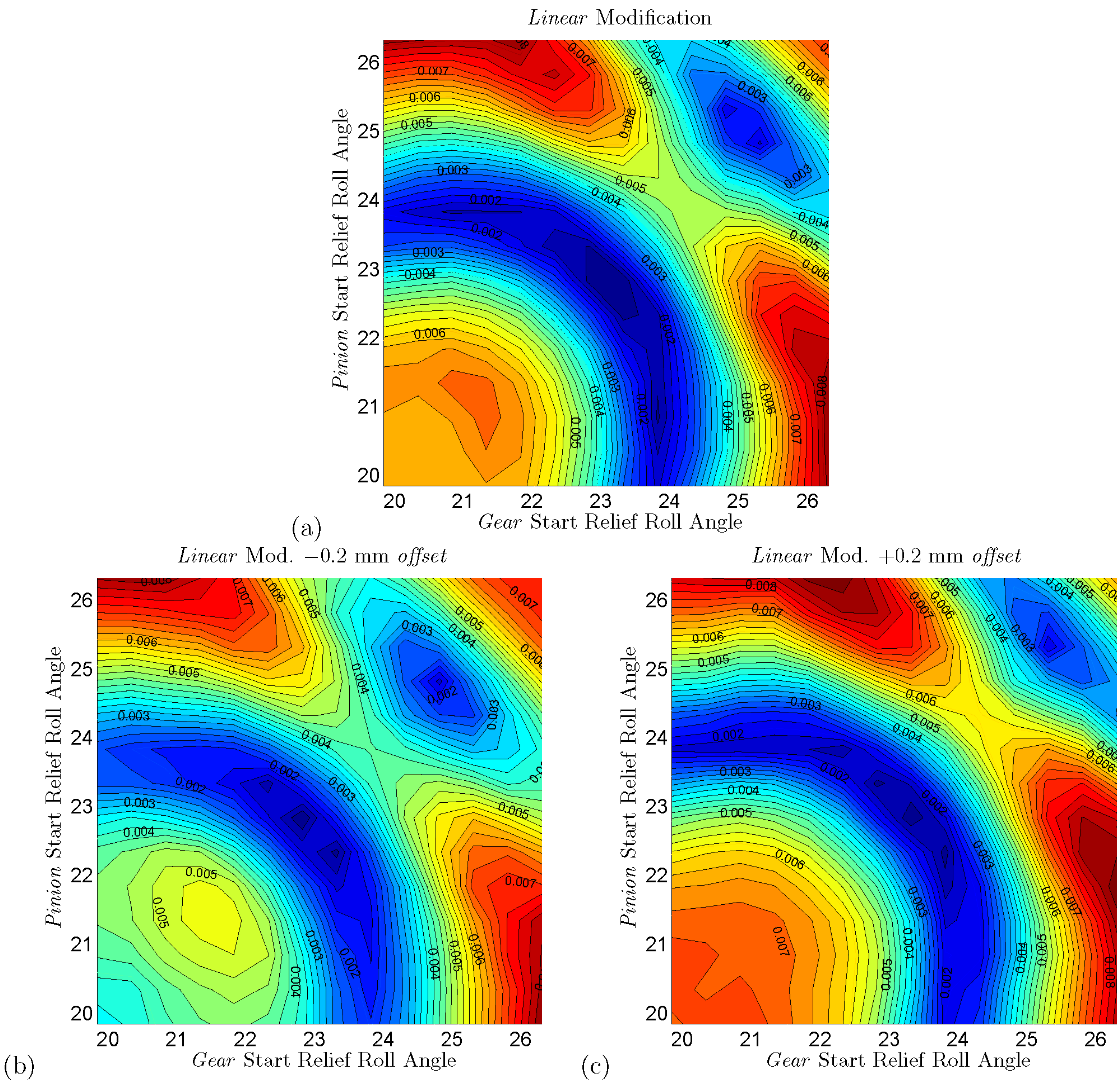


Fig. 10: (a) Linear modification. (b) Linear modification with offset -0.2 mm. (c) Linear modification with offset +0.2 mm.

Contact Pressure anomalies are presented and applied as limit of acceptance on *PPTE* maps. Modifications near to parabolic can show wider acceptance domain, since Edge Contact and strong curvature issues are less dangerous.
Optimum configuration is here found for a gear set at a given load. The new topography introduces a degree of freedom that can be useful in design optimum profile modification for any gear set.

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